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Multi-Scale Based Multifractal Analysis Method

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Abstract. In this paper, the method of caculating multifractal spectrum with partition function is briefly introduced based on multifractal theory. As research object, a growing structure is simulated by computer. Then we discuss the shortage of the partition function method and the effection of free scale zones to multifractal spectrum results. At last, a new approach is presented to express developing surface structure. It can describe the character of developing surface structure more exactly, and will play an important role in analyse and characterization of complex structure.
Keywords: Multifractal, Scale, Partition function.

I. INTRODUCTION

There are a lot of growing phenomenons in nature. It is self-similar in these nature structures and surfaces. Image processing techniques [1, 2] have been used in recognizing and analyzing these nature structrues and surfaces. It is very difficult to describe these structure and surfaces with euclidean geometric knowledge. Fractal [3] and multifractal analysis is an effective method to describe such irregular structure.

Multifractal is a kind of analysis method which can describe and analyse structures or systems from local characteristic to macrocosm [4]. It also describes the changing process of structure or systems. So multifractal method is a useful tool to study on characteristics of nature structures and systems [5]. According to the definition of multifractal, it is not very easy to calculate the multifractal spectrum. But with the help of statistic physics, we can get the multifracal spectrum conveniently by using partition function. When calculating, we usually choose all the data or a relatively straighter line piece at will as free scale zone. In the past, there is not much deep research on how to choose free scale zone. But characteristic of a certain structrue depend on the choice of free scale zone. According to our research, the choice of free scale zone will impact on multifractal spectrum, and there is some important physical meaning related to free scale zone, which is often neglected in the past research. So research on the choice of free scale zone is important for describing the characteristic of structure accurately.

II. THEORY

Let us postulate that P is value of mass distributing in a certain scale. We can relates $P(\varepsilon)$ and ε with the following formula in the process of $\varepsilon \to 0$ [6, 7, 8].

$$P(\varepsilon) \sim \varepsilon^{\alpha} \tag{1}$$

Where α describes the singularity of $P(\varepsilon)$. $N(\varepsilon)$ is the number of $P(\varepsilon)$ with the same α. Multifractal spectrum is defined as follow:

$$N(\varepsilon) \sim \varepsilon^{-f(\alpha)} \tag{2}$$

$$f(\alpha) \sim \frac{\ln[N(\varepsilon)]}{\ln \varepsilon} \tag{3}$$

Where $f(\alpha)$ can be calculated from the slope of $\ln N(\varepsilon) \sim \ln \varepsilon$ curve. The curve of $\alpha \sim f(\alpha)$ is defined as multifractal spectrum. It can describes all the messages of a fractal set.

At present, a partition fuction method is usually used to calculate multifractal spectrum, which can be described as follows:

Devides the image into boxes of $1/\varepsilon \times 1/\varepsilon$,where ε ($\varepsilon = 1/2, 1/2^2, \ldots$) is called scale. $P_{ij}(\varepsilon)$ is caculated as follow:

$$P_{ij}(\varepsilon) = \frac{n_{ij}(\varepsilon)}{\sum n_{ij}(\varepsilon)} \tag{4}$$

where $i, j = 1, 2, \cdots 1/\varepsilon$, $n_{ij}(\varepsilon)$ is the sum of pixels value in a box. According to some formula related to statistical physics, partition function is defined as follow[9,10]:

$$\chi_q(\varepsilon) = \sum P_{ij}(\varepsilon)^q \sim \varepsilon^{-\tau(q)} \tag{5}$$

The value of q is from $-\infty$ to $+\infty$, but in fact, if $|q|$ is large enough, the larger $|q|$

has no effect on multifractal spectrum.

According to Lengend transformation, multifractal is caculated as follows:[8]

$$\alpha = d\tau(q) / d \tag{6}$$

$$f(\alpha) = \alpha \cdot q - \tau(q) \tag{7}$$

III. DISCUSSION ON PARTITION FUNTION METHOD

Many nature structures are self-similar structures, such as microstructures of pyrrhotites[11], soil surface roughness[12], solar wind turbulence[13] and complete genomes [14]. The images of these structures contain multifractal characters. In order to simulate the physical structure of nature object,we build a multifractal image with computer.

Fig.1 is a picture of a kind of structure which grows according to a certain rule by computer simulation.

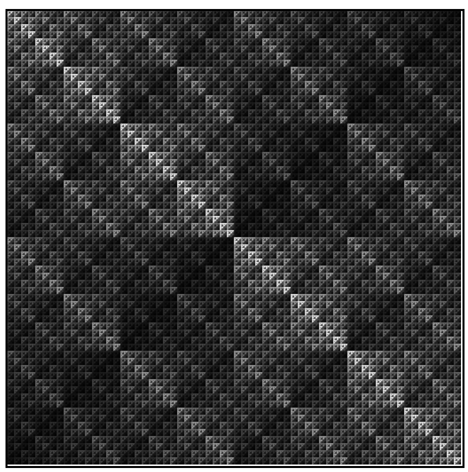

Fig. 1. Structural simulation drawing

According to Eqs. (3)(4), we can calculate the $\ln \chi_q(\varepsilon) \sim \ln(\varepsilon)$ curves (Fig.3) of Fig.1. Because the rule has changed between the 6th, 7th layer, the curve has presented obvious turn at the corresponding position. Analysing the singal curve(when q=–20)， according to the classic method，choose the whole data as the free-scale zone or choose a certain scale in which the curve is more smooth as the free-scale zone, and then calculate $\tau(q)$ (Fig.2). Using the former method, what we calculated is the average slope of the curve(Fig.2 (blue)), the latter is a certain slope of the curve

(Fig.2 (red)). Multifractal spectrum calculated by using the average slope describes "the average rule" of the structure. Obviously, the average rule does not exist in fact. Multifractal spectrum calculated by using a certain slope describes the rule of corresponding layers in the structure. This rule exists, but there also exists another rule which are not described in the structure. So the methods mentioned above can not describe the rule of a structure correctly.

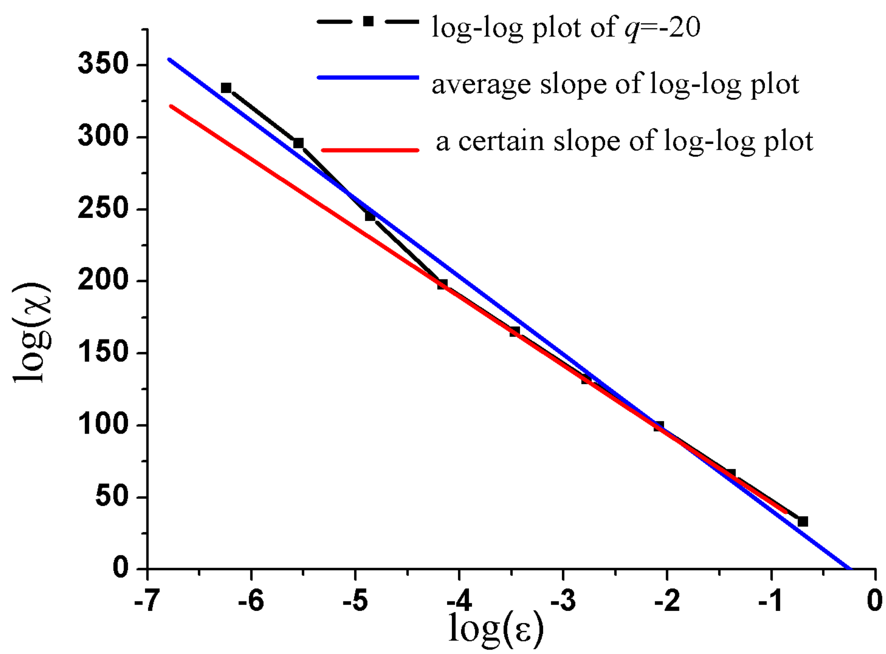

Fig.2. The average slope(blue) and a certain slope(red) of log-log plot when q=-20

IV. THE NEW METHOD OF ANALYSIS ON STRUCTRUE

The classic method can not describe the overall structure and every present rule , so this text proposes a new method, that chooses multi-free-scale zone to calculate multifractal spectrum.

There are obvious slope transitions in curves $\ln \chi_q(\varepsilon) \sim \ln \varepsilon$, so we choose three free scale zones. Tag the zone where $\varepsilon = 1/2, 1/2^2 \cdots 1/2^9$ with the number 1 and where $\varepsilon = 1/2, 1/2^2 \cdots 1/2^6$ with the number 2 and where $\varepsilon = 1/2^7, 1/2^8, 1/2^9$ the number 3. It can be considered that there are macroscopical and microcosmic messages in the zone 2 and 3. With Eqs. (6)(7)(8), we can caculate three multifractal spectrum (Fig.4).

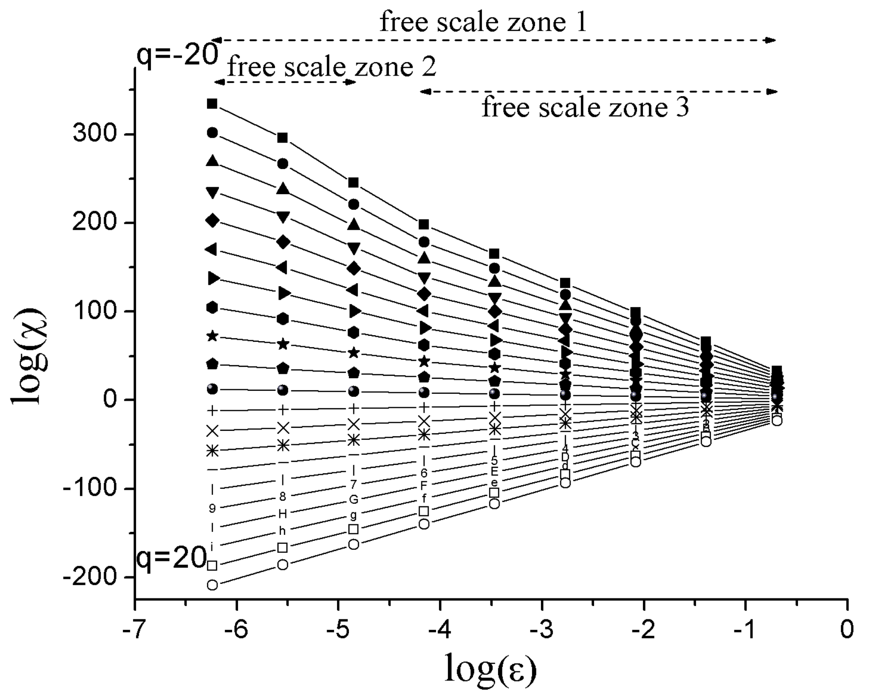

Fig.3. $\log\varepsilon \sim \log\chi$ plot and three free scale zones

In fig.4, spectra A, B, C show separately the multifractal spectrum which are calculated by using zone 1,2,3. There are differences greatly between spectra B and C. It proves that the growth rules of the structure in zone 2and zone 3 differs widely, namely the rules are different between the macroscopic and microcosmic. It is the same with change of structure. Spectra A describe the average rule by using zone 1, which does not exist in the structure obviously. So choosing spectra B and spectra C can describe the growth characteristic of the structure more completely than the only spectra A or the only spectra C.

In a word, during the process of calculating multifractal spectrum of a structure, by choosing different free scale zones, the calculated multifractal spectrum can describe corresponding local characteristics. As to a structure whose rule of growth is single, it is equivalent to choose any free scale zone to calculate multifractal spectrum. The characteristics of this kind of structure can be expressed by one spectra. As to the structure whose growth rule is changeable, we can get the detailed characteristics of different layers by choosing corresponding free scale zone to calculate multifractal spectrum. Only in this way can we describe the multifractal characteristics of structure in an all-round way.

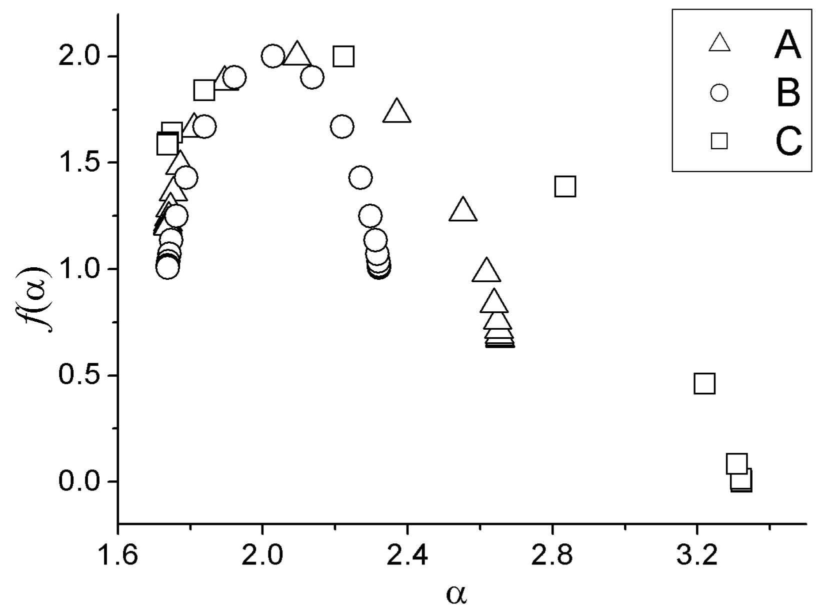

Fig. 4. Multifractal spectrum calculated in three different methods

V. **CONCLUSION**

On the basis of multifractal theory, we analyzed the meanings of partition function method and got the following conclusions:

1. There is some deficiency to use partition function method to calculate multifractal spectrum. It can only describe the average characteristic or a certain existing characteristic of the structure. So the classic method can't get the overall structural characteristic of description.

2. Put forward a new method to express characteristic of growth structure. This method uses multi-free-scale zone to calculate multifractal spectrum of the structure. It can get the growth rule of the structure at each layer, and describe multifractal characteristic of the structure in an all-round way more accurately.

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