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Theoretical Analysis of the Influence of the Thermal Diffusivity of Ceramic Tile on the Thermal Energy Distribution

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Abstract. The influence of the thermal diffusivity of the ceramic tile on the thermal energy distribution was analysed using one dimensional heat equation, which was solved by using method of separation of variables. In the analysis, heat was assumed to be propagated along a rectangular moulded ceramic tile with length (l) and the width being considered negligible with different temperatures ranging from 300^0c to 1300^0c, within a specified time frame. Some parameters such as thermal conductivity, specific heat and mass per unit length of the material were specified. The variation of thermal conductivity and diffusivity with temperature were analysed while that of thermal energy flux u(x,t) variation with position and time for different lengths werealso take into consideration. The distribution of temperature as a function of time for different values of thermal diffusivity was also considered.

Keywords: Thermal diffusivity, thermal energy distribution, heat, ceramic tile, thermal conductivity.Temperature.

INTRODUCTION

The study of the thermophysical properties of engineering materials such as ceramic has been the concern of engineers and scientists as ceramics are gradually dominating in engineering and technology today. Ceramics apart from being used in producing domestic and industrial products such as ceramic wares, pottery, bricks, roofing mica industries, refractories, rocketry; carbides of silicon, boro, tungsten and other elements are used as abrasives and cutting tool materials [1]. The study of the thermal property of the ceramic tile is necessitated by the numerous application area which ceramic materials enjoy such as mining, aero-space, medicine, refinery, food and chemical industries, packaging science, electronics, industrial and transmission electricity, and guided light wave transmission [2].

Thermal conductivity is generated by the movement of electrons and the transfer of lattice vibrations. From the foregoing, it is understood that this property differs for different materials, owing to the atomic structure of the material. In clay soil this property is dependent upon the following factors: texture and mineralogical composition of the soil, water content of the soil. For ceramics, thermal conductivity is affected by factors such as internal porosity, grain boundary and impurities.In heat transfer analysis, thermal diffusivity denoted $\propto$[3] is the thermal conductivity divided by density and specific heat capacity at constant pressure.It has the S.I. unit of m^2/s $\propto = \frac{\boldsymbol{K}}{\rho C_p}$ where $\rho C_p$is called volumetric heat capacity ($J/m^3.k$). In a sense, thermal diffusivity is the measure of thermal inertia[4].In a substance with high thermal diffusivity, heat moves rapidly through because the substance conducts heat quickly relative to its volumetric heat capacity or thermal bulk. Thermal diffusivity is often measured with the flash method [5].It involves heating a strip or cylindrical sample with a short change (reduction in amplitude and phase shift of the pulse) a short distance away (Blumm and [6,7].

Thermal conductivity and diffusivity which are the rate at which heat is dissipated and the measure of transient thermal response of a material to a change in temperature [8], respectively were analyzed by [9] with reference to Abakaliki clay soil and was found that the thermal conductivity of molded clay model affects the energy distribution which was inferred to have resulted from the influence of the density of soil, which is highly affected by the porous nature of the soil. Thermal diffusion (Heat transfer) through materials or substances is studied using heat equation which is a parabolic partial differential equation which describes the distribution of heat or variation in temperature in a given region over time [10]. The transfer equation for thermal energy (Fourier's law), mechanical moment Newton's law of fluid and mass transfer Filck's law of diffusion are similar [11], and analogies among these three processes have been developed to facilitate prediction of conversion from anyone to the other [12].

MATERIALS AND METHOD

Theoretical Method: Consider the schematic representation of idealized physical setting for heat diffusing in a rectangular mould ceramic tile with equation in which the boundary conditions as indicated in the diagram is applied.

u(x, 0) = f(x) (initial temperature distribution)

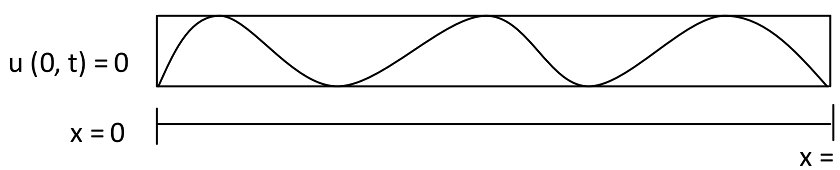


***Fig. 1:** Model of mould ceramic tile with a negligible width*

This equation is solved using the method of separation of variables.

The method applied here for the analysis is the heat equation, which is an important partial differential equation that describes the distribution of heat or variation in temperature, given by

$$\frac{\partial u}{\partial t} = \frac{k}{\sigma p}\frac{\partial^2 u}{\partial x^2} \qquad (1)$$

Where $c^2 = \frac{K}{\sigma \rho} \quad (m^2 s^{-1})$

c^2 = Thermal diffusivity of the material (m^2s^{-1})

k = Thermal conductivity of the material ($Js^{-1}k^{-1}m^{-1}$)

ρ = the density of the material (kgm^{-3})

σ = Specific heat capacity ($JK^{-1}kg^{-1}$)

$\nabla^2 u$ = Laplacian of u.

u = Temperature which is a function of x, y, z over time t. That is, u(x, y, z, t).

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \qquad (2)$$

From Fig 1, we assumed that temperature is oriented along x-axis, that is y=0, z=0, so that heat flows in x-direction only, that is one dimensional heat flow given as

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \qquad (3)$$

SOLUTION TO THE HEAT EQUATION

From equation (3) above signifies one dimensional heat equation

Where $c^2 = \frac{k}{\sigma p}$ ms^{-1}

i. Where themould extends from x=0 to x = l with initial temperature at the end maintained at zero.

Consideration the boundary condition as specified below

u(0, t) = 0, u(l,t)=0 for all t>0

u(x,0) = f(x)

By applying the concept of separation of variable equation 3 becomes

$$X T^1 = c^2 TX^{11} \qquad (4)$$

Resulting to

$$\frac{1}{c^2}\frac{T^1}{T} = \frac{X^{11}}{X} \tag{5}$$

If k = - P^2

Then (6) $\frac{T^1}{c^2T} = \frac{X^{11}}{X} = -p^2$

$X^{11} + p^2X = 0$ (7)

$T^1 + p^2c^2T = 0$ (8)

Therefore, $m^2X + p^2X = 0$ (9)

$(m^2 + p^2)X = 0$

Since $X \neq 0$, then

$m^2 + P^2 = 0$

$m^2 + p^2 = 0 \Rightarrow m^2 = -p^2$

$\Longrightarrow m = \pm ip$

$$\Rightarrow u(x) = C\ell^{im\,x} + D\ell^{-mx} = C\ell^{ipx} + D\ell^{-ipx} \tag{10}$$

But $\left.\begin{aligned} C\ell^{ipx} &= C\cos px + iC\sin px \\ D\ell^{-ipx} &= D\cos px - iD\sin px \end{aligned}\right\}$ (3. (11)

Putting equation (10) in (11)

We have, $u(x) = C\cos px + iC\sin px + D\cos px - iD\sin px$

$$(C + D)\cos px + i(C - D)\sin px$$

Let A = (C + D), B= i(C-D) since they are both arbitrary constants.

$$U(x) = A\cos px + B\sin px \tag{12}$$

For that of equation 8 the solution become

$$T(t) = \ell^{-c^2P^2t} \cdot \ell^{k} \qquad (13)$$

But ℓ^{k} is constant, therefore, $\ell^{k} = C$

$$\therefore \; T(t) = C\ell^{-c^2P^2t} \qquad (14)$$

thus, the general solution becomes

$$U(x,t) = XT = \{A\cos px + B\sin px\} C\ell^{-P^2c^2t} \qquad (15)$$

If we let $Pc = \lambda$, $p = \dfrac{\lambda}{c}$, we now obtain

$$U(x,t) = \left\{A cos\frac{\lambda}{c}x + B sin\frac{\lambda}{c}x\right\} C\ell^{-\lambda^2t} \qquad (16)$$

Using our earlier specified boundary conditions,

$$\therefore \; u(x,t) = \sum_{n=1}^{\infty} \left\{ B_n \sin\frac{n\pi x}{l} \right\} \ell^{-\lambda^2t} \qquad (17)$$

$$\lambda = \frac{n\pi c}{l} \qquad (18)$$

To get B_n, we use Fourier series technique (B. S. Rajput, 2010).

Which results in

$$u(x,t) = \frac{2}{l}\sum_{n=1}^{\infty}\left\{\left[\int_0^l \sin\frac{n\pi x}{l} f(x)\,dx\right]\ell^{-\lambda^2t}\sin\frac{n\pi x}{l}\right\} \qquad (19)$$

Where $\lambda_n = \dfrac{nc\pi}{l}$ n= 1, 2, 3,....

RESULTS AND DISCUSSION

The general solution of the heat equation that was obtained is given in equation (19)

That is

$$u(x,t) = \frac{2}{l}\sum_{n=1}^{\infty}\left\{\left[\int_0^l \sin\frac{n\pi x}{l} f(x)\,dx\right]\ell^{-\lambda^2 t}\sin\frac{n\pi x}{l}\right\}$$

From this equation, the analysis on ceramic tile modeled as depicted in figure 1 was carried out.

Considering the temperature ranging from 300°C to 1300°C using 100°C interval, at time ranging from 0 to 120 seconds. The analysis was carried on the thermal diffusivity(c) measured in (m^2s^{-1}).

Figure 2 represents graph of thermal conductivity against temperature. While figure3 represents thermal diffusivity against temperature. These graphs have decrease in values of both thermal conductivity and diffusivity with increase in temperature. That is at room temperature, the value of thermal conductivity of the ceramic tile is 39w/mk and/or slightly above. While that of thermal diffusivity is $0.12x10^{-4}m^2/s$.

Figure 4 depicts the graph of temperature distribution against time when thermal diffusivity (c) = $0.12m^2s^{-1}$ and length of the material (l) = 0.18m, while Figure 5 is the graph of temperature distribution against time when thermal diffusivity (c) = $0.15m^2s^{-1}$ and length of the material (l) = 0.18m.

The two graphs clearly depicted exponential decay of temperature as the time increases. This shows that length and time determine the temperature distribution on ceramic tile which in turn affects the thermal diffusivity of the material.

Figure 6 to 11 showed the graph of energy distribution as a function of time. In figures 6, 7, 8 and 9, there is an exponential decrease in energy distribution as the time increases. While10 and 11 showed exponential increase of the thermal energy distribution.

Thethermal energy distribution as observed in figure 12 to 16 behaved differently in a manner that appeared to be periodic and gradually attenuated to zero as the time increases which mighty be as a result of exponential factor of the function. The frequency of the distribution is more intense with x =-1 to 1 meaning that as the length increases the intensity of the distribution decreases exponentially.

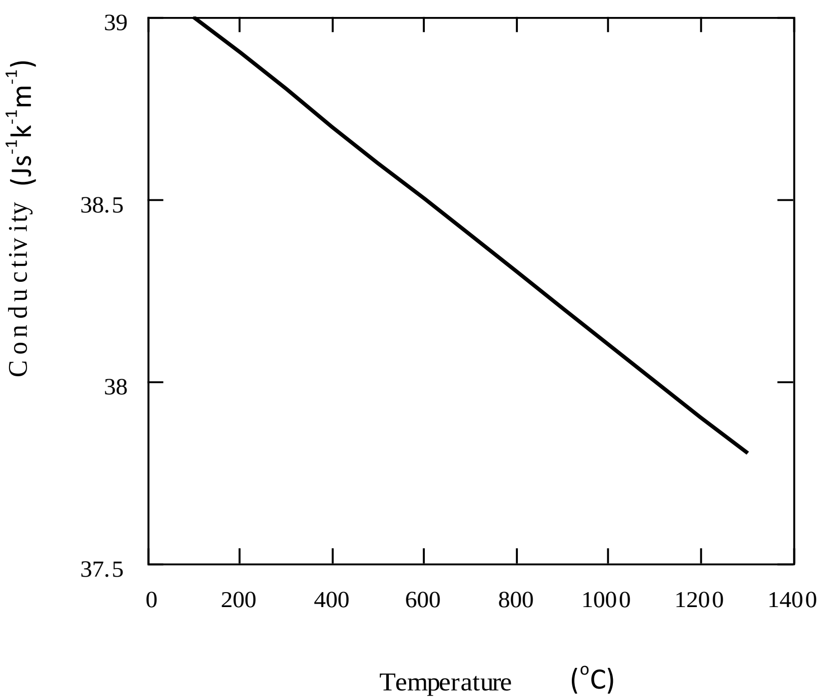


Fig. 2:*Graph of thermal conductivity against temperature*

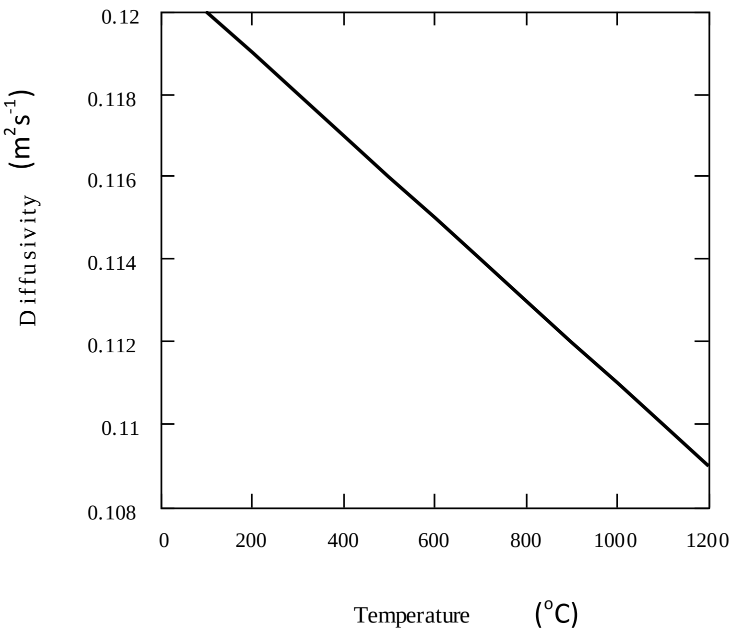


Fig. 3: *Graph of thermal diffusivity against temperature*

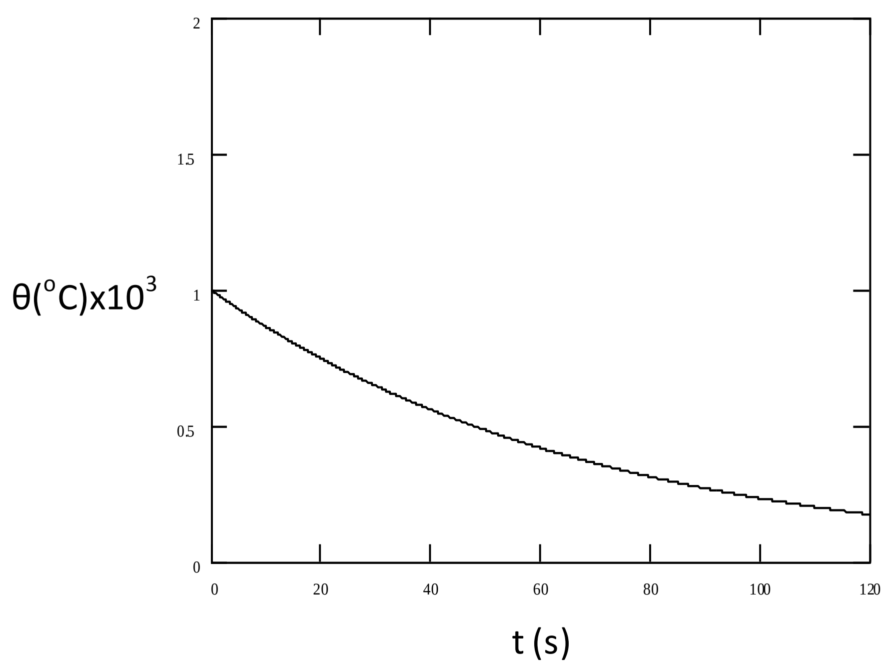


Fig. 4: *Graph of temperature distribution against time when*

$c = 0.12\ m^2s^{-1}\quad L = 0.18m$

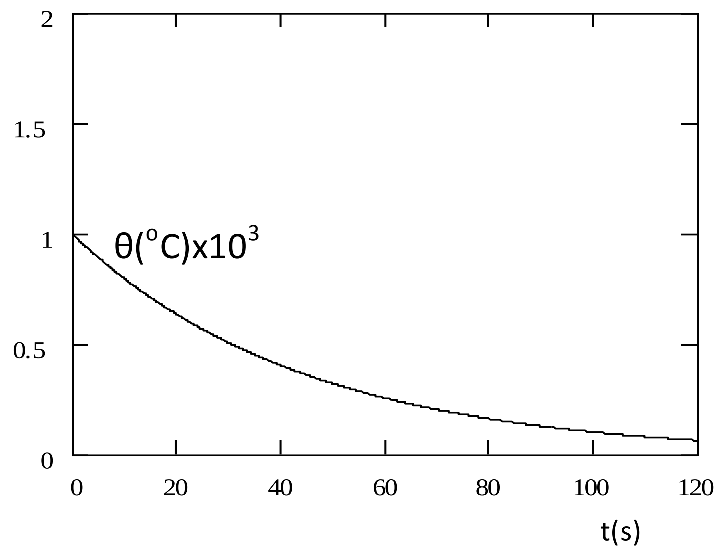


Fig. 5: *Graph of temperature distribution against time when*

$c = 0.15\ m^2s^{-1}\quad L = 0.18m$

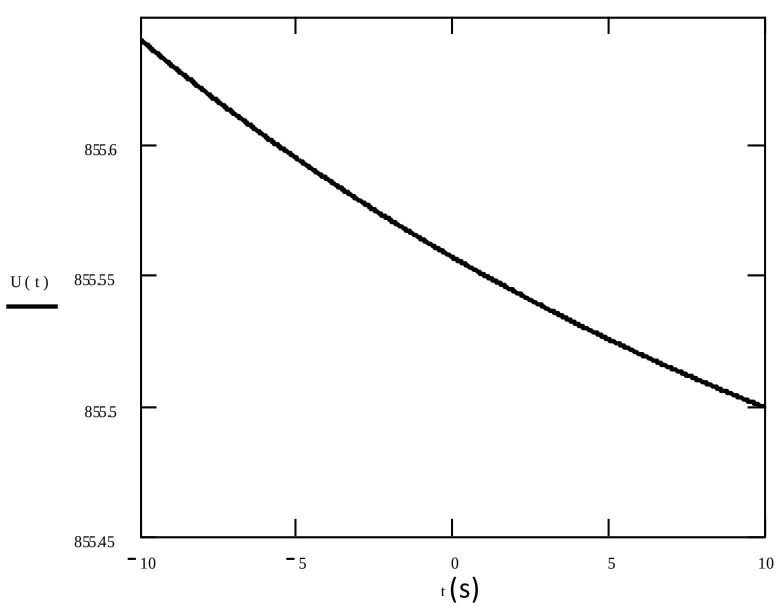


Fig.6: *Graph of energy distribution as a function of time for*

$c = 1.10\times10^{-5}m^2s^{-1}$

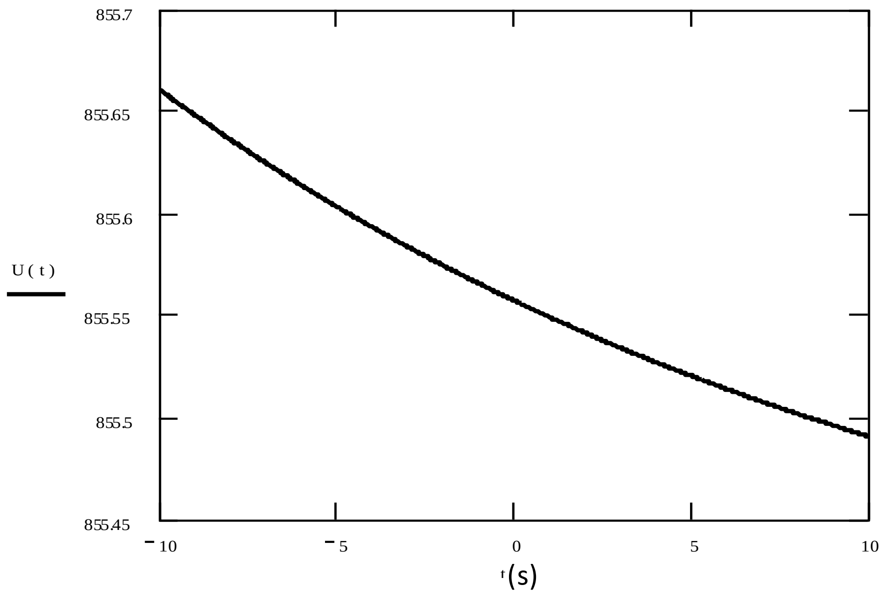


Fig. 7:_Graph of energy distribution as a function of time for_

$c = 1.2\times10^{-5}\, m^2 s^{-}$

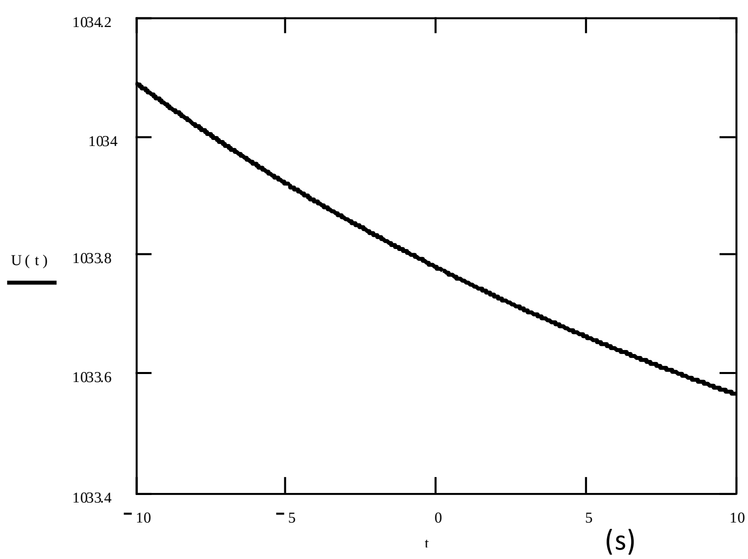


Fig.8: _Graph of energy distribution as a function of time when_

$c = 1.10\times10^{-5}\, m^2 s^{-1}$ _and_ $x = 0.132m$

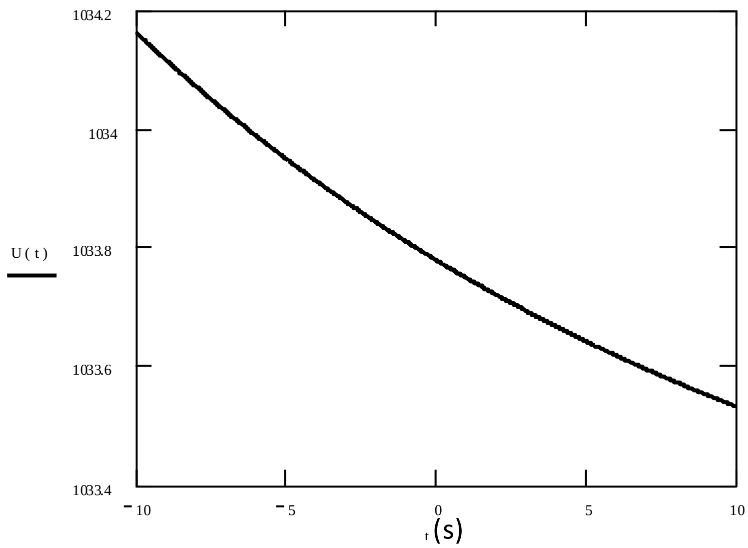


Fig.9:*Graph of energy distribution as a function of time when*

$c = 1.2\times10^{-5} m^2 s^{-1}$ and $L= 0.18m$ $x = 0.132m$

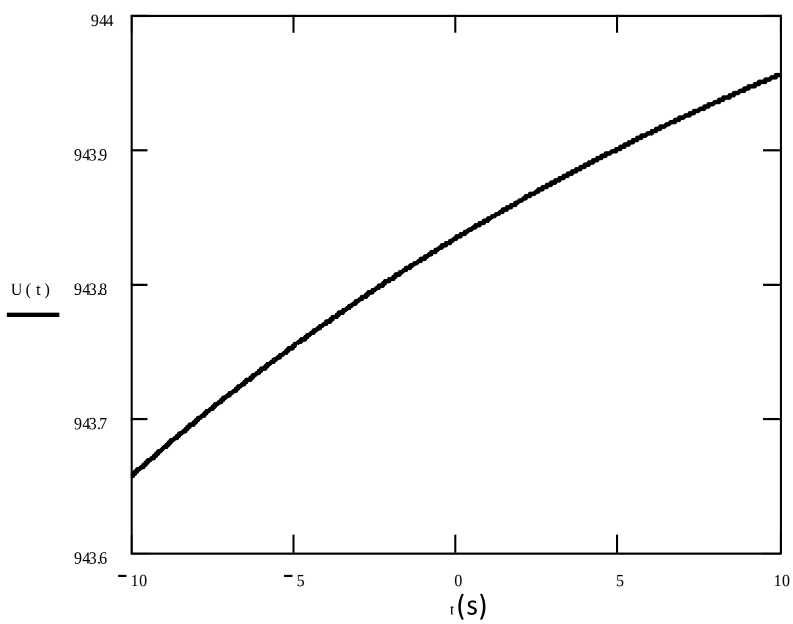


Fig. 10:*Graph of energy distribution as a function of time when*

$c = 1.10\times10^{-5} m^2 s^{-1}$ and $x = 0.116m$

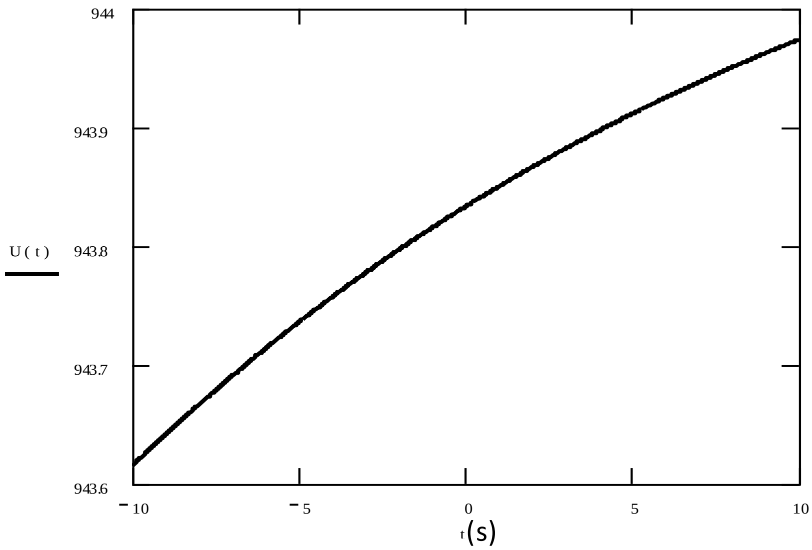
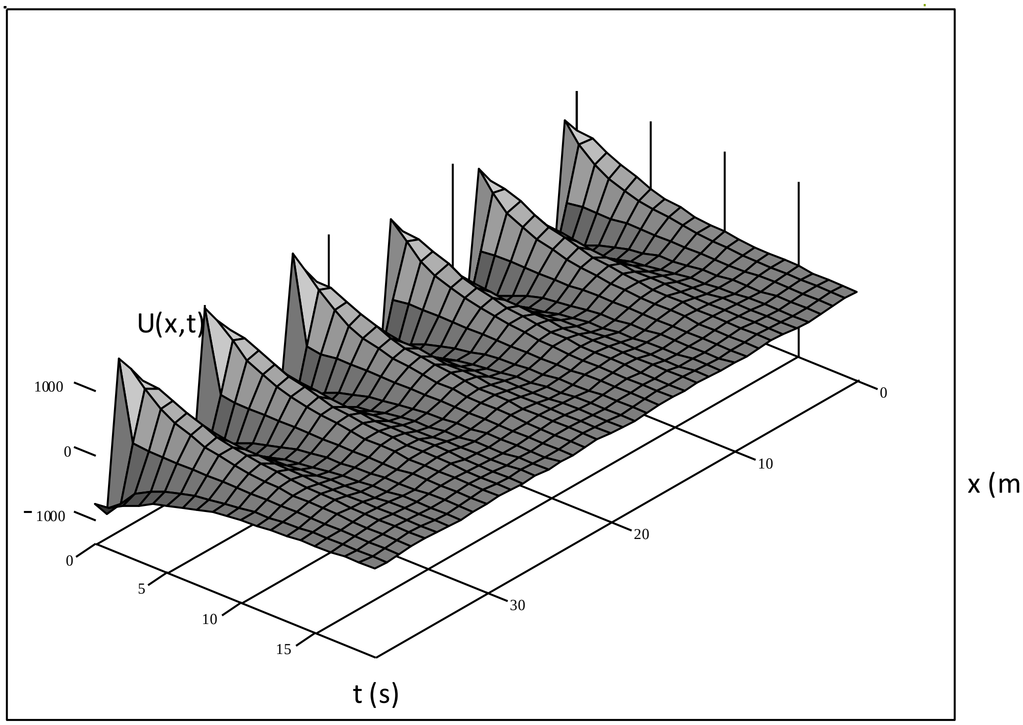


Fig. 11:*Graph of energy distribution as a function of time when*

$c = 1.2\times10^{-5}\ m^2s^{-1}$ *and* $x = 0.116m$



M

Fig.12: *Thermal energy distribution plotted against position and time with thermal diffusivity of 0.12 m^2s^{-1}.*

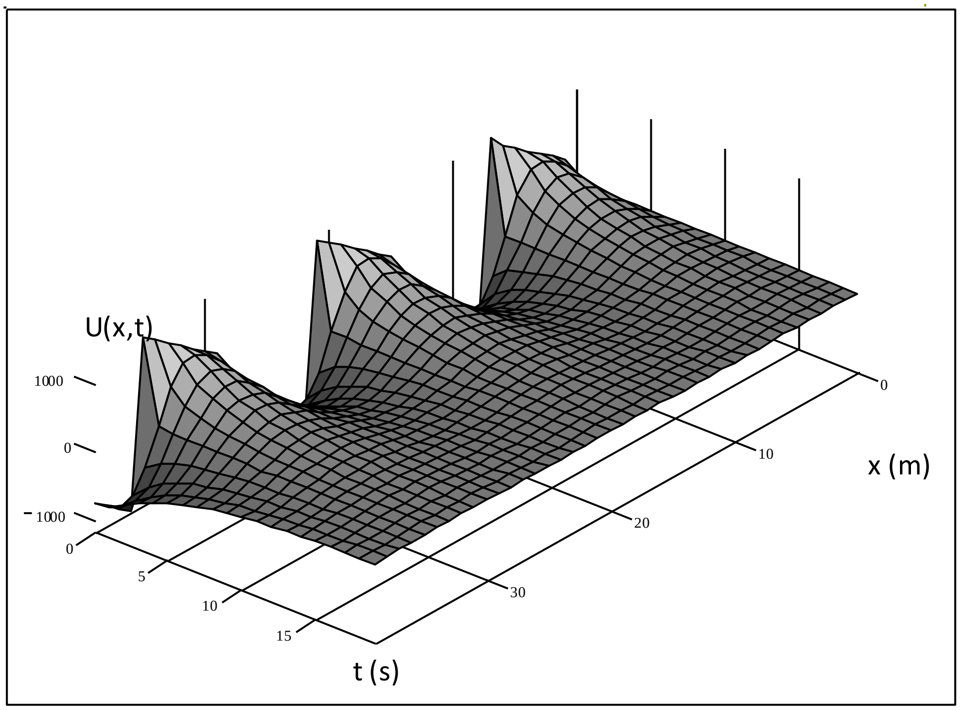


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Fig.13: *Thermal energy distribution plotted against position and time with thermal diffusivity of 0.12 m^2s^{-1}.*

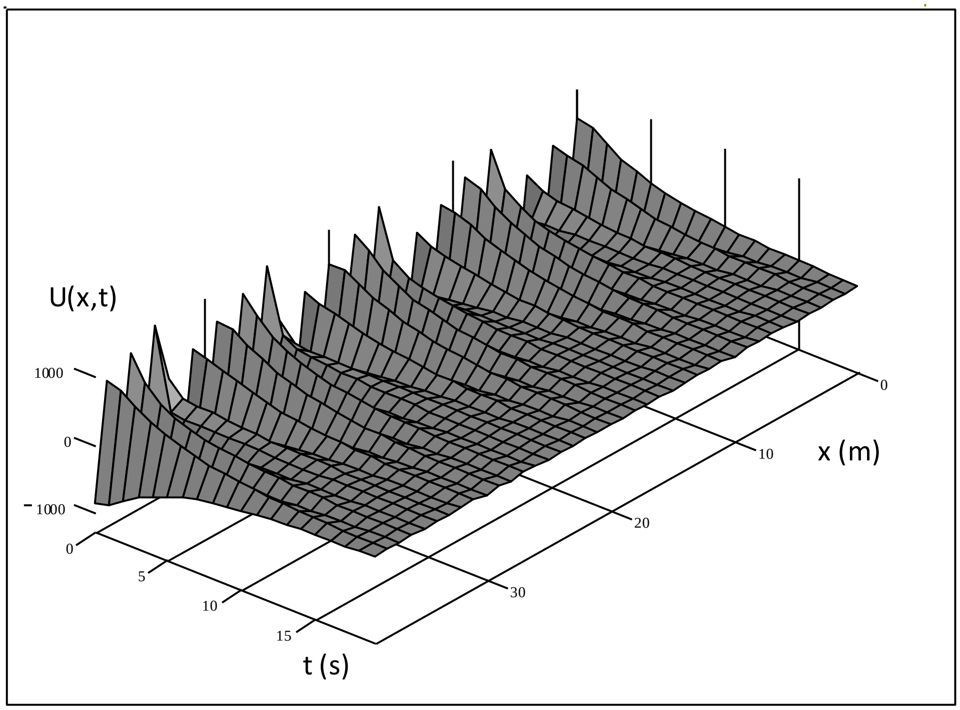

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Fig .14: *Thermal energy distribution plotted against position and time with thermal diffusivity of 0.12* m^2s^{-1}.

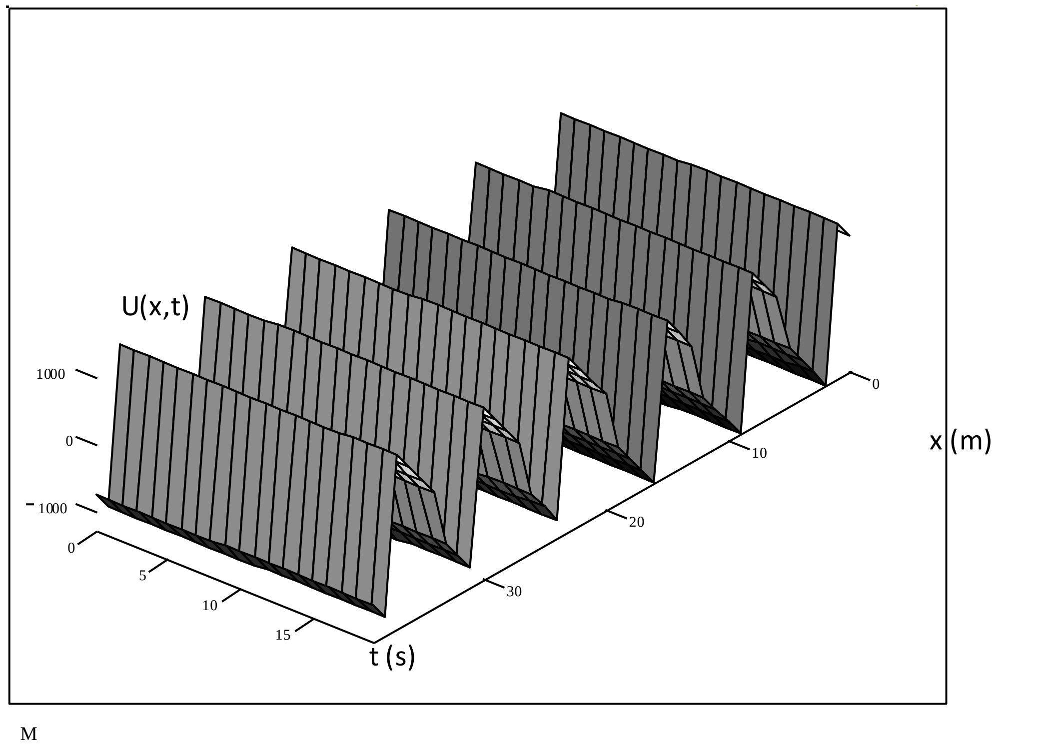

Fig.15: *Thermal energy distribution plotted against position and time with thermal diffusivity of* $0.12\times10^{-4}\ m^2s^{-1}$.

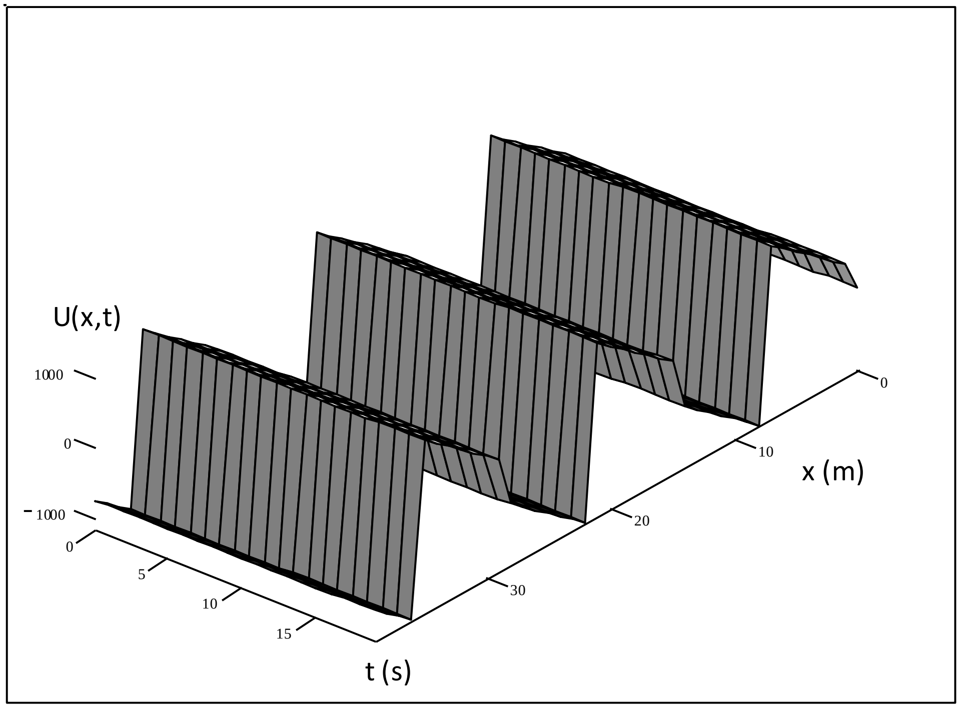

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Fig.16: *Thermal energy distribution plotted against position and time with thermal diffusivity of* $0.12\times10^{-5}\ m^2s^{-1}$.

CONCLUSION

The concept of one dimensional heat equation was used in the analysis of the influence of the thermal diffusivity of ceramic tile on the energy distribution profile. It was found that the energy distribution along modeled ceramic tile is affected by thermal diffusivity of the material. It was also observed that energy distribution maintained periodic profile along the length and showed highest peak when the time was minimal (zero) but as the time increased, the energy distribution started flattening to zero which is in accordance. Thus, thermal energy distribution is a function of time with unique behaviour as a result of variation in thermal diffusivity as considered in our analysis.

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