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Study of Electromagnetic Wave Propagation in Periodic Dielectric Structure; MathCAD Analysis

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Abstract. The propagation of electromagnetic waves in periodic dielectric structure was analyzed using plane wave equation of a periodic electric field in conjunction with the application of the Schrodinger's equation as an operator with Mathieu's differential equation. The relationship between electric field (E), width of dielectric slabs, b, period, a, transverse electric mode $\mathbf{k_y}$ and wave number were obtained. Graphically, it was shown that different values of the ratio of the width of the dielectric slabs, b to the period, a affect the wave propagation. However, the behaviours of electromagnetic waves propagation through a dielectric medium depend on the solid state properties of the medium. Also, how electromagnetic waves propagate as well as its basic properties was presented.

Key words: Electromagnetic Waves, Dielectric Constant, Periodic Structure, Dielectric Slabs, Period, Electric field, Wave number. Propagation

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I. INTRODUCTION

In recent time, scientists have been interested in the way and manner wave behave as it propagate through materials of various solid state properties and as such have voraciously working to know the materials that offer best conditions to harness the optimum application for the propagating wave [1,2]. Electromagnetic waves propagation through a periodic structure is one of the many interesting problems in mathematical physics and has been of interest for materials for optoelectronics, solar cell, etc.

Electromagnetic wave incident on a Crystal at normal or oblique on the surface of crystal

l experiences attenuation as the wave propagate through [3]. The extent of the attenuate suffered by the wave depends on the solid state properties of the crystal such as the dielectric function of the crystal which is related to the refractive index, in which properties of the solid state crystal materials strongly depend on[4].

The study of the propagation of wave in the periodic medium has a complex discrete composition which makes it difficult to set up the boundary condition for atoms forming the structure of the crystal [5]. Though dielectric structure does not conduct but on the application of electric field, the dielectric tires to behave like that of non-dielectric ones as the positive and negative ions in dielectric structure form a polarity in the presence of electric field [6].

Also, it has been shown that the refractive index of dielectric crystal is real and slightly less than unit, giving waves which propagate without attenuation with a phase velocity slightly lesser than the velocity of the light in vacuum [7].

Crystal of different materials compound are being grown, developed and characterized and studied in order to identify the nature of the refractive index and dielectric functions.

Various properties such as refractive index, frequency limit and dielectric constants are part of the properties of dielectric crystal when wave are allowed to propagate through it.

The analysis of scalar wave behaviour on the rough surface was studied by [8] Also, the propagation of a plane electromagnetic wave through flat surface is extended to multiple scattering theory development by [9].

The research on the behaviour of an incident wave over a rough surface of crystal has been studied by Feinberg who obtained effective impedance of the interface coupled with easier studies on scattering and multiple scattering as have been proposed by[10].

In this research, electromagnetic wave propagation in a periodic dielectric medium was analyzed using plane wave equation of periodic electric field with the application of Schrodinger's equation as an operator with Mathieu's differential equation.

The objective of the study is to analyze the effect of periodic dielectric structure as electromagnetic wave propagates through it. The research work explains the analysis of electromagnetic waves propagation in one-dimensional periodic dielectric structure.

II. Solution of plane wave through a one-dimensional periodic media.

A numerical solution to the exact eigenvalue equation for the one-dimensional periodic problem is obtained with the help of MathCAD software.

Consider a one-dimensional periodic array of dielectric slabs with period a and dielectric insert width b is

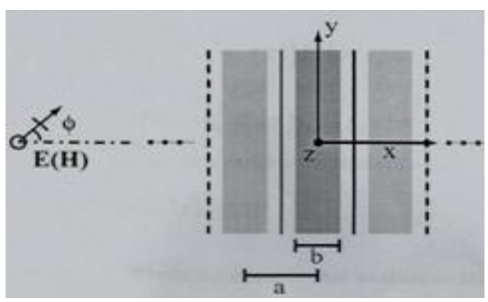


Fig.1; Dielectric Structure model with incident wave normally on it.

One-dimensional lattice of dielectric slabs of width **b** in a periodic lattice with period **a**.The electric field in a periodic structure shown is given by

$$E(x, y) = \hat{z} E_z(x, y) = zE_p(x,)e^{-jK_{xo}x}e^{-jK_y y} \qquad (1)$$

where E_x is the periodic electric field that propagates only in the xy plane, i.e. $k_z = 0$, and K_{xo} is a prescribed propagation constant, without loss of generality.

Since the electric field must satisfy the wave equation, we apply the operator; $(\nabla^2_{xy} + k^2)$ to $E_z(x, y)$ in (1) above, observing that the dielectric constantis a function of x.

We have that, $\nabla_{xy}^{2} E_{z}(x, y)+k_{0}^{2} \varepsilon r(x) E_{z}(x, y)=0$ (2)

For $k^{2}=k_{0}^{2}$

ε_{r} = dielectric constant insert

Assuming the parallel slabs are infinite in the y and Z directions, equation (2) can be simplified

But $\nabla_{xy}^{2}=\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}$ (3)

Substituting (3) into (2)

We have that

$$\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}\right) E_{z}(x,y)+k_{0}^{2} \varepsilon_{r}(x) E_{z}(x,y)=0 \quad (4)$$

$$\frac{\partial^{2}}{\partial x^{2}} E_{z}(x,y)+\frac{\partial^{2}}{\partial y^{2}} E_{z}(x,y) k_{0}^{2} \varepsilon_{r}(x) E_{z}(x,y)=0 \quad (5)$$

But $\mathrm{E}(x, y)=\widehat{z} E_{z}(x, y)=\widehat{z} E_{p}(x,) e^{-jK_{xo}} x e^{-jK_{y}} y$ (6)

i.e. $\mathrm{E}(x, y)=\hat{Z} E_{p}(x,) e^{-j\left(k_{x0} x+k_{y} y\right)}$ (7)

Substituting (7) into (5) and differentiate twice with respect to y

We have that

$$\frac{\partial^{2}}{\partial x^{2}} E_{z}(x,y)-k_{y}^{2} E_{z}(x,y)+k_{0}^{2} \varepsilon_{r}(x) E_{z}(x,y)=0 \quad (8)$$

$$\frac{\partial^{2}}{\partial x^{2}} E_{z}(x,y)-k_{y}^{2} E_{z}(x,y)=-k_{0}^{2} \varepsilon_{r}(x) \varepsilon_{r}(x) E_{z}(x,y)=0 \quad (9)$$

Multiplying -1 to both sides we have that

$$-\frac{\partial^{2}}{\partial x^{2}} E_{z}(x,y)-k_{y}^{2} E_{z}(x,y)+k_{y}^{2} E_{z}(x, y)=k_{0}^{2} \varepsilon_{r}(x) E_{z}(x,y)=0 \quad (10)$$

This equation can be written as

$$-\frac{\partial^{2}}{\partial x^{2}} E_{z}(x,y)+\left(k_{y}^{2}-k_{0}^{2} \varepsilon_{r}(x)\right) E_{z}(x,y)=0 \quad (11)a$$

Or $$\frac{\partial^{2}}{\partial x^{2}} E_{z}(x,y)-\left(k_{y}^{2}-k_{0}^{2} \varepsilon_{r}(x)\right) E_{z}(x,y)=0$$

(11)b

The above differential equation is similar to the canonical form of Mathieu's differential equation given by;

$$\frac{\partial^2 y}{\partial x^2} + [a + 2q\cos(2x)]y = 0 \qquad (12)$$

If we substitute u = ix to the above equation, then we obtain the Mathieu's modified differential equation

$$\frac{\partial^2 y}{\partial x^2} - [a + 2q\cosh(2u)]y = 0 \qquad (13)$$

According to Bloch's theorem for fixed values of *a, q Mathieu's* equation admits a complex valued solution of the form

$$f(a,q,x) = \exp(i\mu x)(a,q,x) \quad (14)$$

Here μ is a complex number called the Mathieu exponent, and *p* is a complex valued function which is periodic in x

In equation (1 1) above we will approximate the dielectric $\varepsilon_r(x)$ by a periodic function.

$\varepsilon_r(x) = \cosh\left(\frac{2\pi bx}{a}\right)$ which is periodic with period 2π and $\frac{b}{a} = 1$

Thus we wish to solve the ordinary differential equation.

We use hyperbolic cosine because of equation (13).

$$\frac{\partial^2}{\partial x^2}E_z(x,y) - \left(k_y^2 - k_0^2\cosh\left(\frac{2\pi bx}{a}\right)\right)E_z(x,y) = 0 \qquad (15)$$

For the purpose of obtaining a numerical solution we vectorize the above equation as follows;

$E_z(x,y) = y_1$

$$y_2 = \frac{\partial y_1}{\partial x} = \frac{\partial^2 y_1}{\partial x^2} = \frac{\partial}{\partial x}E_z(x,y) \Rightarrow \frac{\partial y_2}{\partial x} = \frac{\partial^2}{\partial x^2}E_z(x,y) \qquad (16)$$

Hence equation (15) becomes

$$\frac{\partial y_1}{\partial x} = y_2$$

$$\frac{\partial y_2}{\partial x}\left(k_y^2 - k_0^2 \cosh\left(\frac{2\pi bx}{a}\right)\right) y_1 \qquad (17)$$

For the numerical solution we take, $k_y^2 - k_0^2 = 1$, and $\frac{b}{a} = 0.5$, $\frac{b}{a} = 0.3545$, $\frac{b}{a} = 1$

MATCAD SOLUTION

$$D(t, y) = \begin{bmatrix} Y_1 \\ 1 - \cosh(2.\pi.0.5)\ )Y_0 \end{bmatrix} \qquad (18)$$

Derivative rector

Define additional arguments for the ODE solver:

$t_0 = 0$ Initial value of independent variable

$t_1 = 2.\pi$ Terminal value of independent variable

$Y_0 = \begin{bmatrix} 0.5 \\ 2 \end{bmatrix}$ Vector of initial function values

N= 1000 Number of solution values on $[t_0, t_1]$

Solution matrix:

$M = Rkadapt\ (Y_0, t_0, t_1, N, D)$

$t = M^{<0>}$ Independent variable values

$E = M^{<1>}$ First solution function values

$D = M^{<2>}$ Second solution function values

III. RESULTS AND DISCUSSION

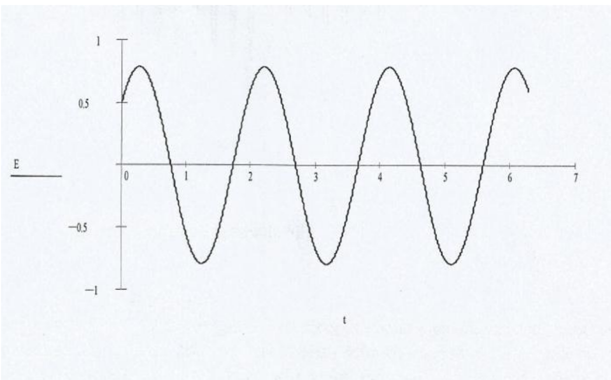
When electromagnetic wave impinge on a periodic dielectric medium, the relationship between Electric field (E), transverse electric mode k_y,wave number k_0, width of the dielectric slabs, b, and period, a was obtained from this equation.

$$\frac{\partial y_2}{\partial x}\left(k_y^2 - k_0^2 \cosh\left(\frac{2\pi bx}{a}\right)\right)y_1$$

$$y_2 = \frac{\partial y_1}{\partial x}$$

On the relationship, some values were obtained for electric field, width of the dielectric slabs and period for the wave propagation for $k_y^2 = k_0^2 = 1$ for numerical solution so as to know how electromagnetic waves propagating through periodic medium with different values of the ratio of the width of the dielectric slabs, b to the period a varies with the wave propagation along x – direction.

From Fig.1 and fig. 2, it was observed that when the values of $\frac{b}{a} = 0.3545$ and 0.5, respectively for the wave propagating through the periodic dielectric medium, the oscillating frequency is low while in case of the value $\frac{b}{a} = 1$ As shown in the Fig.4, the frequency of the wave appears higher than in first two cases.



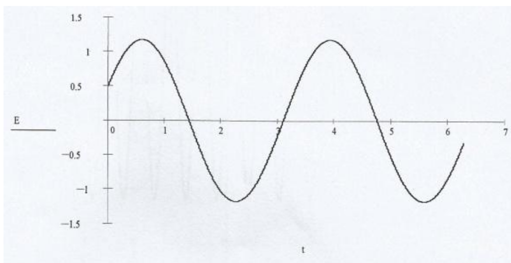


Fig. 2: Wave profile when b/a = 0.8Fig. 3; Wave profile when b/a =0.3545

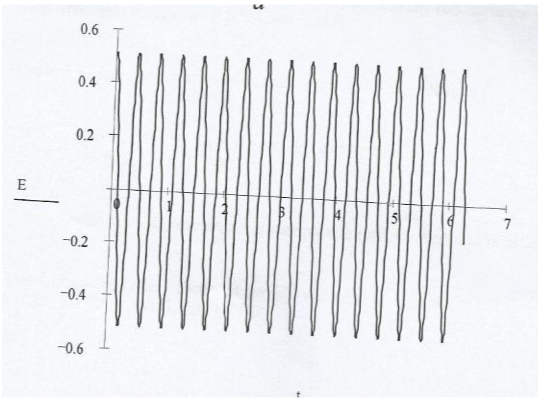


Fig. 4: Wave profile when $b/a = 1.0$

IV.CONCLUSION

In general, the analysis of the propagation of electromagnetic waves in periodic dielectric structure using plane wave equation of a periodic electric field and Mathieu's differential equation had been carried out numerically using MathCAD from where relationship between electric field, dielectric constant, wave number, width of dielectric slabs and the period of the wave propagation was established from the solution.

From the graphs, it was seen that the behaviour of electromagnetic wave propagation through a dielectric structure depends on the size of the slab in conjunction with the solid state properties of the structure such as the dielectric constant as enshrined in the solution also which as manifested in the profile of the propagated wave shown in the graphs for different values of the ratio of the width of the dielectric slabs,

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