Communications in Applied Sciences
ISSN 2201-7372
Volume 6, Number 1, 2018, 1-10

INFINITY PRESS
www.infinitypress.info

Radial Solutions of the Schrodinger's Equation with Superposition of Yukawa and Manning-Rosen Applying the Nikiforov-Uvarov Method

Benedict I. Ita[1*], Hitler Louis[1*], Muyiwa M. Orosun[2], Nelson Nzeata-Ibe[1], Alexander I. Ikeuba[1], and Thomas O. Magu[1]

[1]Physical/Theoretical Chemistry Research Group, Department of Pure and Applied Chemistry, Faculty of Physical Sciences, University of Calabar, Calabar, CRS, Nigeria.

[2]Department of Physics, University of Ilorin. Ilorin, Kwara State, Nigeria.

*Correspondences: iserom2001@yahoo.com (B.I. Ita) and louismuzong@gmail.com (H. Louis)

Radial Solutions of the Schrodinger's Equation with Superposition of Yukawa and Manning-Rosen Applying the Nikiforov-Uvarov Method

Abstract

The solutions of the Schrödinger equation with Manning-Rosen plus Yukawa potential (MRYP) have been presented using the Pekeris-like approximation of the Coulomb term and parametric Nikiforov-Uvarov (NU) method. The bound state energy eigenvalues and the corresponding un-normalized eigenfunctions are obtained regarding Jacobi polynomials. Also, Yukawa, Manning-Rosen, and Coulomb potentials have been recovered from the mixed potential, and their eigenvalues obtained.

Keywords: Schrödinger equation, Manning-Rosen potential, Yukawa potential, Pekeris-like approximation, ParametricNikiforov-Uvarov method, Jacobi polynomials

1 INTRODUCTION

In quantum mechanics, one of the exciting problems is to obtain exact solutions of the Schrödinger equation. To do this, a real potential is customarily chosen to derive the energy eigenvalues and the eigenfunctions of the Schrödinger equation (Magu *et al.,* 2017). These solutions describe the particle dynamics in non-relativistic quantum mechanics (Antia *et al.,* 2015). Several authors have studied the bound states of the Schrödinger equation using different potentials and methods. Some of these potentials play significant roles in many fields of Physics such as Molecular Physics, Solid State and Chemical Physics (Ita *et al*., 2016). The Manning-Rosen potential has been intensely studied and applied in quantum systems and Yukawa potential, and its classes have been considered in Schrodinger formalism (Louis *et al.,* 2016).

The purpose of the present paper is to solve the Schrödinger equation for the mixed potential MRYP using the parametric NU method. The paper is organized as follows: After a brief introduction in section 1, the NU method is reviewed in section 2. In section 3, we solve the radial Schrödinger equation using the NU method. Finally, we discuss our results in section 4and a brief conclusion is then advanced in section 5.

2 NIKIFOROV-UVAROV METHOD

The Nikiforov-Uvarov (NU) method is based on the solutions of a generalized second-order linear differential equation with special orthogonal functions. The Schrödinger equation and Schrödinger-like equations of the type as:

$$\psi''(r) + [E - V(r)]\psi(r) = 0, \quad (1)$$

can be solved by this method. To do this equation (1) is transformed into equation of hypergeometric type with appropriate coordinate transformation $s = s(r)$ to get

$$\psi''(s) + \frac{\bar{\tau}(s)}{\sigma(s)}\psi'(s) + \frac{\bar{\sigma}(s)}{\sigma^2(s)}\psi(s) = 0, \quad (2)$$

To solve equation (2) we can use the parametric NU method. The parametric generalization of the NU method is expressed by the generalized hypergeometric type equation [19]

$$\psi''(s) + \frac{(c_1 - c_2 s)}{s(1 - c_3 s)}\psi'(s) + \frac{1}{s^2(1 - c_3 s)^2}[-\epsilon_1 s^2 + \epsilon_2 s - \epsilon_3]\psi(s) = 0, \quad (3)$$

where$\sigma(s)$ and $\bar{\sigma}(s)$ are polynomials atmost second degree, and $\bar{\tau}(s)$ is a first degree polynomial. The eigenfunctions (equation 4) and corresponding eigenvalues (equation 5) to the equation become

$$\psi(s) = N_n s^{c_{12}}(1 - c_3 s)^{-c_{12} - \frac{c_{13}}{c_3}} P_n^{\left(c_{10} - 1, \frac{c_{11}}{c_3} - c_{10} - 1\right)}(1 - 2c_3 s), \quad (4)$$

$$(c_2 - c_3)n + c_3 n^2 - (2n + 1)c_5 + (2n + 1)\left(\sqrt{c_9} + c_3\sqrt{c_8}\right) + c_7 + 2c_3 c_8 + 2\sqrt{c_8 c_9} = 0, \quad (5)$$

Where

$$c_4 = \frac{1}{2}(1 - c_1), c_5 = \frac{1}{2}(c_2 - 2c_3), c_6 = {c_5}^2 + \epsilon_1, c_7 = 2c_4 c_5 - \epsilon_2, c_8 = {c_4}^2 + \epsilon_3 \;,\; c_9 = c_3 c_7 + {c_2}^2 c_8 + c_6,\; c_{10} = c_1 + 2c_4 + 2\sqrt{c_8}, c_{11} = c_2 - 2c_5 + 2\left(\sqrt{c_9} + c_3\sqrt{c_8}\right), c_{12} = c_4 + \sqrt{c_8},$$

$$c_{13} = c_5 - \left(\sqrt{c_9} + c_3\sqrt{c_8}\right), \quad (6)$$

$N_n$is the normalization constant and $P_n^{(\alpha,\beta)}$are the Jacobi polynomials.

3. SOLUTIONS OF THE RADIAL PART OF SCHRÖDINGER EQUATION WITH MRYP POTENTIAL

The radial Schrödinger equation (for which $l \neq 0$) is given as

$$\frac{d^2 R_{nl}(r)}{dr^2} + \frac{2\mu}{\hbar^2}\left[E - V(r) - \frac{\lambda\hbar^2}{2\mu r^2}\right] R_{nl}(r), \quad (7)$$

Where $\lambda = l(l+1)$ and $V(r)$ is the potential energy function. However in this paper we solving for the s-wave Schrodinger equation for which $l = 0$. Thus, the Manning-Rosen potential (MRP) is given as (Louis *et al.*, 2016)

$$V(r) = -\left[\frac{Ce^{-\alpha r} + De^{-2\alpha r}}{(1 - e^{-\alpha r})^2}\right] \quad (8)$$

The Yukawa potential (YP) is given as (Antia *et al.*, 2015)

$$V(r) = -\frac{V_o e^{-\alpha r}}{r}, \quad (9)$$

where V_0 is the potential depth of the YP and α is an adjustable positive parameter. In equation (8)

$C$and D are constants. The sum of these potentials known as MRYP is given as

$$V(r) = -\left[\frac{Ce^{-\propto r}+De^{-2\propto r}}{(1-e^{-\propto r})^2}\right] - \frac{V_0 e^{-\propto r}}{r} \tag{10}$$

Making the transformation $s = e^{-\alpha r}$equation (10) becomes

$$V(s) = -\left[\frac{CS+DS^2}{(1-S)^2}\right] - \frac{\propto V_0 S}{1-S} \tag{11}$$

Again, applying the transformation $s = e^{-\alpha r}$ to get the form that NU method is applicable, equation (7) gives a generalized hypergeometric-type equation as

$$\frac{d^2R(s)}{ds^2} + \frac{(1-s)}{(1-s)s}\frac{dR(s)}{ds} + \frac{1}{(1-s)^2 s^2}[-(\beta^2 - F + B)s^2 + (2\beta^2 + A + B)s - (\beta^2)]R(s) = 0, \tag{12}$$

Where

$$-\beta^2 = \frac{2\mu E}{\alpha^2\hbar^2}, A = \frac{2\mu C}{\alpha^2\hbar^2}, B = \frac{2\mu V_0}{\alpha\hbar^2}, F = \frac{2\mu D}{\alpha^2\hbar^2}, \frac{1}{r} \approx \frac{\propto}{(1-e^{-\propto r})} \approx \frac{\propto}{(1-S)}, \tag{13}$$

Comparing equation (12) with equation (3) yields the following parameters

$$c_1 = c_2 = c_3 = 1, c_4 = 0, c_5 = -\frac{1}{2}, c_6 = \frac{1}{4} + \beta^2 + B - F, c_7 = -2\beta^2 - A - B, c_8 = \beta^2, c_9 = \frac{1}{4} - (A+F), c_{10} = 1 + 2\sqrt{\beta^2}, c_{11} = 2 + 2\left(\sqrt{\frac{1}{4} - A - F} + \sqrt{\beta^2}\right), c_{12} = \sqrt{\beta^2}, c_{13} = -\frac{1}{2} - \left(\sqrt{\frac{1}{4} - A - F} + \sqrt{\beta^2}\right), \epsilon_1 = \beta^2 + B - F, \epsilon_2 = 2\beta^2 + A + B, \epsilon_3 = \beta^2, \tag{14}$$

Now using equations (5), (13) and (14) we obtain the energy eigenspectrum of the MRYP as

$$\beta^2 = \left[\frac{A+B-\left(n^2+n+\frac{1}{2}\right)-(2n+1)\sqrt{\frac{1}{4}-A-F}}{(2n+1)+2\sqrt{\frac{1}{4}-A-F}}\right]^2, \tag{15}$$

Equation (15) can be solved explicitly and the energy eigen spectrum of MRYP becomes

$$E = -\frac{\alpha^2\hbar^2}{2\mu}\left\{\left[\frac{\frac{2\mu C}{\propto^2\hbar^2}+\frac{2\mu V_0}{\alpha\hbar^2}-\left(n^2+n+\frac{1}{2}\right)-(2n+1)\sqrt{\frac{1}{4}-\frac{2\mu C}{\propto^2\hbar^2}-\frac{2\mu D}{\alpha^2\hbar^2}}}{(2n+1)+2\sqrt{\frac{1}{4}-\frac{2\mu C}{\propto^2\hbar^2}-\frac{2\mu D}{\alpha^2\hbar^2}}}\right]^2\right\}, \tag{16}$$

We now calculate the radial wave function of the MRYP as follows

The weight function $\rho(s)$ is given as [19]

$$\rho(s) = s^{c_{10}-1}(1 - c_3 s)^{\frac{c_{11}}{c_3}-c_{10}-1}, \tag{17}$$

Using equation (14) we get the weight function as

$$\rho(s) = s^U (1 - s)^V, \tag{18}$$

Where $U = 2\sqrt{\beta^2}$ and $V = 2\sqrt{\frac{1}{4} - A - F}$

Also, we obtain the wave function $\chi(s)$ as [19]

$$\chi(s) = P_n^{c_{10}-1, \frac{c_{11}}{c_3}-c_{10}-1}(1 - 2c_3 s). \tag{19}$$

Using equation (14) we get the function χ(s) as

$$\chi(s) = P_n^{(U,V)}(1 - 2s). \tag{20}$$

Where $P_n^{(U,V)}$ are Jacobi polynomials

Lastly,

$$\varphi(s) = s^{c_{12}}(1 - c_3 s)^{-c_{12}-\frac{c_{13}}{c_3}}. \tag{21}$$

And using equation (14) we get

$$\varphi(s) = s^{U/2}(1 - s)^{V-1/2}. \tag{22}$$

We then obtain the radial wave function from the equation [19]

$$R_n(s) = N_n \varphi(s) \chi_n(s). \tag{23}$$

As

$$R_n(s) = N_n s^{U/2}(1 - s)^{(V-1)/2} P_n^{(U,V)}(1 - 2s). \tag{24}$$

Where n is a positive integer and N_n is the normalization constant.

4 DISCUSSION

We have solved the radial Schrödinger equation and obtained the energy eigenvalues for the Manning-Rosen plus Yukawa potential (MRYP) in equation (16).

The following cases are considered:

Case 1: If $C = D = 0$ in equation (10), the potential turns back into the Yukawa potential and equation (16) yields the energy eigen values of the Yukawa potential as

$$E = -\frac{\hbar^2}{2\mu}\left[\frac{\frac{2\mu V_0}{\hbar^2} - \alpha^2(n+1)^2}{2(n+1)}\right]^2, \quad (25)$$

Equation (25) is similar to equation (30) obtained by Antia *et al*., 2015

Case 2: If $\alpha \to 0, V_0 = Ze^2$ in equation (25), the energy eigen values for Coulomb potential becomes

$$\mathrm{E} = -\frac{Z^2 e^4 \mu}{2\hbar^2 n'^2} \quad (26)$$

Where $n' = n + 1$ in this case.

Case 3: If $V_0 = 0$ the potential in equation (10) yields the Manning-Rosen potential with energy eigen values given as

$$E = -\frac{\alpha^2\hbar^2}{2\mu}\left\{\left[\frac{\frac{2\mu C}{\alpha^2\hbar^2} - \left(n^2+n+\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - \frac{2\mu C}{\alpha^2\hbar^2} - \frac{2\mu D}{\alpha^2\hbar^2}}}{(2n+1) + 2\sqrt{\frac{1}{4} - \frac{2\mu C}{\alpha^2\hbar^2} - \frac{2\mu D}{\alpha^2\hbar^2}}}\right]^2\right\} \quad (27)$$

Eq. (27) is also similar to Manning-Rosen potential bound state energy obtained by Louis *et al*., 2016

5 CONCLUSIONS:

We have obtained the energy eigenvalues and the corresponding un-normalized wave function using the parametric NU method for the Schrödinger equation with MRYP. Special cases of the potential have also been considered. The results of the numerical analysis are shown in **Table 1**. The potential decreases with an increase in screening parameter α. Furthermore, the negative value of the eigenenergy states depicts that the eigensolutions are bound. This work can be extended for the spectroscopic study of atom and molecules.

References

Antia, A.D, Essien, E. B Umoren and C. C Eze, "Approximate Solution of the non-relativistic Schrodinger Equation with Inversely Quadratic Yukawa Plus Mobius Square Potential Via Parametric Nikiforov-Uvarov Method," *Advances in Physics Theories and Application*, Vol. 44, (2015)

Ita, B.I, A.I Ikeuba, O. Obinna, "Solution of Schrödinger Equation with Inversely Quadratic Yukawa potential plus Woods-Saxon potential using the parametric Nikiforov-Uvarov method," *Journals of Advance in Physics*, Vol. 18, No. 2, pp. 2094-2098 (2015).

Ita, B.I, and A. I. Ikeuba, "Solutions to the Klein-Gordon Equation with Inversely Quadratic Yukawa plus Inversely Quadratic Potential using Nikiforov-Uvarov Method," *Journals of theoretical physics and cryptography*, Vol. 8 (2015)

Ita, B.I, Nyong, N.O. Alobi, H. Louis and T.O. Magu (2016): Bound State Solution of the Klein-Gordon Equation for Modified Echart Plus Inverse Square Molecular Potential with Improved new Approximation Scheme to Centrifugal Team. Equatorial Journal of Computational and Theoretical Sciences, Volume 1, Issue 1, (2016), pp. 55-64.

Louis, H, Ita, B.I, Nyong, , T.O Magu, N.O Alobi and N.A Nzeata-ibe, "Approximate Solution of the N-Dimensional radial Schrodinger equation for Kratzer plus reduced Pseudoharmonic Oscillator potential within the frame work of N-U Method" J. of NAMP., Vol. 36, No. 2, pp 199-204 (2016)

Louis, H, Ita, B.E Nyong, T.O Magu, N.A Nzeata-ibe and S. Barka, "Radial Solution of the s-wave D-Dimensional Non-relativistic Schrodinger equation for Generalized Manning-Rosen plus Mie-type Nuclei potential within the framework of Nikiforov-Uvarov Method" *J. of NAMP.*, Vol. 36, No. 2, pp 193-198 (2016)

Magu, T.O. Ita, B.I, Nyong, B.E, H. Louis, (2017): Radial solution of the s-wave Klein-Gordon, the equation for generalised wood-Saxon plus Mie-type potential using Nikiforov-Uvarov J. Chem. Soc. Nigeria, Vol. 41, No. 2, pp21-26 [2017].

Table 1

Energy eigenvalues $E(eV)$ of the MRYP potential for ђ=μ=1, $V_0 = 0.2$, C = -0.1, D = 0.1 with different α values.

n	$\alpha = 0.01$	$\alpha = 0.1$	$\alpha = 2$	$\alpha = 5$
1	−12.05405	−0.1249999	−1.852812	−12.05405
2	− 5.384668	− 0.086805	− 4.351249	− 27.67680
3	−3.050450	−0.08	− 7.850703	− 49.551012
4	− 1.970112	− 0.0840500	−12.350450	−77.675648
5	−1.383338	−0.093889	−17.850312	− 112.050449
6	−1.029612	− 0.107780	− 24.350229	− 152.675330
7	−0.800110	−0.1250000	− 31.850175	−199.550253