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Effects of radiation and viscous dissipation on unsteady free convective flow past a moving vertical porous plate embedded in a porous medium

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Abstract

Effects of radiation on unsteady free convection flow of a viscous incompressible fluid past a moving vertical plate embedded in a porous medium on taking viscous dissipation into account have been studied. The governing partial differential equations have been solved numerically by applying a Crank - Nicolson's type of implicit finite difference method with a tri-diagonal matrix manipulation and an iterative procedure. The variations of the fluid velocity and temperature are presented graphically. It is found that the radiation decelerates the fluid velocity. The fluid velocity increases with an increase in Darcy number. An increase in Eckert number leads to rise in fluid velocity and temperature. Further, it is found that the magnitude of the shear stress at the plate increases with an increase in either radiation parameter or Prandtl number. The rate of heat transfer at the plate increases with an increase in radiation parameter. This model finds applications in geophysics, astrophysics and also in the design of high temperature industrial processing systems.

Key words: Free convection, radiation, Prandtl number, Grashof number, Darcy number, porous medium and viscous dissipation.

1 Introduction

The heat transfer from different geometries embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and underground energy transport. A very significant area of research in radiative heat transfer, at the present time is the numerical simulation of combined radiation and convection /conduction transport processes. The effort has arisen largely due to the need to optimize industrial system such as furnaces, ovens and boilers and the interest in our environment and in non-conventional energy sources such as the use of salt-gradient solar ponds for energy collection and storage. In particular, natural convection induced by the simultaneous action of buoyancy forces resulting from thermal diffusion is of considerable interest in nature and in many industrial applications such as geophysics, oceanography, drying processes and solidification of binary alloy. Convective heat transfer in porous media has received considerable attention in recent years owing to its importance in various technological applications such as fibre and granular insulation, electronic system cooling, cool combustors, oil extraction, thermal energy storage and flow through filtering devices, porous material regenerative heat exchangers. Books by Nield and Bejan [1], Bejan and Kraus [2] and Ingham et al. [3] excellently describe the extent of the research information in this area. The viscous dissipation effects are important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. In the literature, extensive research work is available to examine the effect of natural convection on flow past a plate. Callahan and Manner [4] first considered the transient free convection flow past a semi infinite plate by explicit finite difference method. Unsteady free convective flow on taking into account the mass transfer phenomenon past an infinite vertical porous plate with constant suction was studied by Soundalgekar and Wavre [5]. Kafousias et al. [6] have studied the effects of free convective currents on the flow field of an incompressible viscous fluid past an impulsively started infinite vertical porous plate with constant suction. However, this analysis is not applicable for other fluids whose Prandtl number is different from unity. Soundalgekar and Ganesan [7] have analyzed transient free convective flow past a semi infinite vertical flat plate, taking into account mass transfer by an implicit finite difference method of Crank-Nicolson type. Free convection at a vertical plate with transpiration has investigated by Kolar and Sastri [8]. Yih [9] have analyzed the effect of transpiration on coupled heat and mass transfer in mixed convection over a vertical plate embedded in a saturated porous medium. Elbashbeshy [10] has investigated the mixed convection along a vertical plate embedded in non-darcian porous medium with suction and injection. Chin et al. [11] has studied the effect of variable viscosity on mixed convection boundary layer flow over a vertical surface embedded in a porous medium. MHD steady free convection flow from vertical surface in porous medium has been studied by Reddy [12]. Thermal radiation in fluid dynamics has become a significant branch of the engineering sciences and is an essential aspect of various scenarios in mechanical, aerospace, chemical, environmental, solar power and hazards engineering. For some industrial applications such as glass production and furnace design and in space technology applications such as cosmical flight aerodynamics rocket, propulsion systems, plasma physics and spacecraft re-entry aerothermodynamics which operate at higher temperatures, radiation effects can be significant. In view of this, Hossain and Takhar [13] have analyzed the effect of radiation on mixed convection along a vertical plate with uniform surface temperature. Pal and Talukdar [14] have studied the buoyancy and chemical reaction effects on MHD mixed convection heat and mass transfer in a porous medium with thermal radiation and ohmic heating. Vasu et al. [15] have studied the radiation and mass transfer effects on transient free convection flow of a dissipative fluid past semi-infinite vertical plate with uniform heat and mass flux. Gundagani [16] has obtained a finite element solution of thermal radiation effect on unsteady MHD flow past a vertical porous plate with variable suction. In most of the studies mentioned above, viscous dissipation is neglected. Gebhart [17] has shown the importance of viscous dissipative heat in free convection flow in the case of isothermal and constant heat flux at the plate. Gebhart and Mollendorf [18] have considered the effects of viscous dissipation for external natural convection flow over a surface. Soundalgekar [19] has analyzed viscous dissipative heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate. Maharajan and Gebhart [20] have reported the influence of viscous dissipation effects in natural convective flows, showing that the heat transfer rates are reduced by an increase in the dissipation parameter. Israel Cookey et al. [21] have investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Srihari et al. [22] have studied MHD free convection flow of an incompressible viscous dissipative fluid in an infinite vertical oscillating plate with constant heat flux. Suneetha et al. [23] have analyzed the effects of viscous dissipation and thermal radiation on hydromagnetic free convective flow past an impulsively started vertical plate. Suneetha et al.[24] have studied the effects of thermal radiation on the natural convective heat and mass transfer of a viscous incompressible gray absorbing-emitting fluid flowing past an impulsively started moving vertical plate with viscous dissipation. Ahmed and Batin [25] have obtained an analytical model of MHD mixed convective radiating fluid with viscous dissipative heat. Babu et al. [26] have studied the radiation and chemical reaction effects on an unsteady MHD convective flow past a vertical moving porous plate embedded in a porous medium with viscous dissipation. Kishore et al. [27] have analyzed the effects of thermal radiation and viscous dissipation on MHD heat and mass diffusion flow past an oscillating vertical plate embedded in a porous medium with variable surface conditions.

The objective of the present work is to study the effects of radiation on unsteady free convective flow of a viscous incompressible fluid past a moving vertical plate embedded in a porous medium on taking viscous dissipation into account. At time $t \leq 0$, both the fluid and plate are at rest with constant temperature T_∞. At time $t > 0$, the plate at $y = 0$ starts to move in its own plane with a velocity u_0 and the plate temperature is raised to T_w. The governing equations have been solved numerically using Crank- Nicolson's method. It is found that the fluid velocity u decreases with an increase in radiation parameter Ra. It is observed that the fluid velocity u increases with an increase in either Darcy number Da or Eckert number Ec. It is also found that the fluid temperature θ decreases with an increase in radiation parameter Ra. Further, it is found that the absolute value of the shear stress τ_x at the plate $(\eta = 0)$ increases with an increase in either Ra or Ec. The rate of heat transfer $-\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}$ at the plate $(\eta = 0)$ decreases with an increase in Ra.

2 Formulation of the problem and its solution

Consider the unsteady hydrodynamic flow of a viscous incompressible radiative fluid past a moving vertical plate embedded in a porous medium on taking into account viscous dissipation. The x-axis is taken along the vertical plate in an upward direction and y-axis is taken normal to the plate (see Fig.1). At time $t \leq 0$, both the fluid and plate are at rest with constant temperature T_∞. At time $t > 0$, the plate at $y = 0$ starts to move in its own plane with a velocity u_0 and T_w is the plate temperature . It is also assumed that the radiative heat flux in the x- direction is negligible as compared to that in the y- direction. As the plate is infinitely long, the velocity and temperature fields are functions of y and t only.

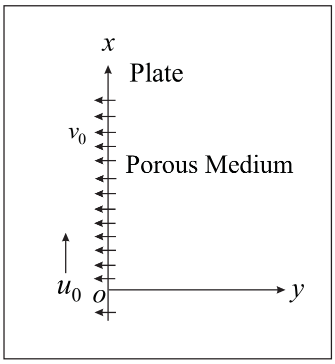

Fig.1: Geometry of the problem

Under the above assumptions and on using the usual Boussinesq approximation, the governing Navier-Stokes equations and energy can be written as

$$\frac{\partial u'}{\partial t} - v'_0\frac{\partial u'}{\partial y} = \nu\frac{\partial^2 u'}{\partial y^2} + g\beta(T - T_\infty) - \frac{\nu}{k^*}u', \quad (1)$$

$$\rho c_p \left(\frac{\partial T}{\partial t} - v_0' \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u'}{\partial y} \right)^2 - \frac{\partial q_r}{\partial y}, \tag{2}$$

where u' is the fluid velocity in the x-direction, T the fluid temperature, g the acceleration due to gravity, β the coefficient of thermal expansion, μ the coefficient of viscosity, ν the kinematic viscosity, ρ the fluid density, k^* permeability of the porous medium, k the thermal conductivity, c_p the specific heat at constant pressure and q_r the radiative heat flux.

The initial and boundary conditions are

$$\begin{aligned}
&u' = 0, \;\; T = T_\infty \;\text{ for all }\; y \;\text{ and }\; t \le 0, \\
&u' = u_0, \;\; T = T_w \;\text{ at }\; y = 0 \;\text{ at }\; t > 0, \\
&u' \to 0, \; T \to T_\infty \;\text{ as }\; y \to \infty \;\text{ for }\; t > 0.
\end{aligned} \tag{3}$$

It has been shown by Cogley et al.[28] that in the optically thin limit for a non-gray gas near equilibrium, the following relation holds

$$\frac{\partial q_r}{\partial y} = 4(T - T_\infty) \int_0^\infty K_{\lambda_0} \left(\frac{\partial e_{\lambda p}}{\partial T} \right)_0 d\lambda, \tag{4}$$

where K_λ is the absorption coefficient, λ is the wave length, $e_{\lambda p}$ is the Planck's function and subscript $'0'$ indicates that all quantities have been evaluated at the temperature T_∞ which is the temperature of the plate at time $t \le 0$. Thus, our study is limited to small difference of plate temperature to the fluid temperature.

On the use of the equation (4), equation (2) becomes

$$\rho c_p \left(\frac{\partial T}{\partial t} - v_0' \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u'}{\partial y} \right)^2 - 4 \left(T - T_\infty \right) I, \tag{5}$$

where

$$I = \int_0^\infty K_{\lambda_0} \left(\frac{\partial e_{\lambda p}}{\partial T} \right)_0 d\lambda. \tag{6}$$

Introducing the non-dimensional variables

$$\eta = \frac{u_0 y}{\nu}, \quad \tau = \frac{t\, u_0^2}{\nu}, \quad u = \frac{u'}{u_0}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \tag{7}$$

equations (1) and (5) become

$$\frac{\partial u}{\partial \tau} - S \frac{\partial u}{\partial \eta} = Gr\, \theta + \frac{\partial^2 u}{\partial \eta^2} - \frac{1}{Da} u, \tag{8}$$

$$Pr \left(\frac{\partial \theta}{\partial \tau} - S \frac{\partial \theta}{\partial \eta} \right) = \frac{\partial^2 \theta}{\partial \eta^2} + Pr Ec \left(\frac{\partial u}{\partial \eta} \right)^2 - Ra\, \theta, \tag{9}$$

where $Da = \frac{k^* u_0^2}{\nu^2}$ is the Darcy number, $S = \frac{v_0'}{u_0}$ the suction parameter, $Ra = \frac{4I T_\infty}{k}$ the radiation parameter, $Pr = \frac{\rho \nu c_p}{k}$ the Prandtl number, $Gr = \frac{g \beta \nu (T_w - T_\infty)}{u_0^3}$ the Grashof number and $Ec = \frac{u_0^2}{c_p(T_w - T_\infty)}$ the Eckert number.

The corresponding boundary conditions for $u(\eta, \tau)$ and $\theta(\eta, \tau)$ are

$$\begin{aligned} &u = 0, \;\theta = 0 \;\text{ for }\; \eta \geq 0 \;\text{ and }\; \tau \leq 0, \\ &u = 1, \;\; \theta = 1 \;\text{ at }\; \eta = 0 \;\text{ for }\; \tau > 0, \\ &u \to 0, \;\theta \to 0 \;\text{ as }\; \eta \to \infty \;\text{ for }\; \tau > 0. \end{aligned} \tag{10}$$

3 Numerical Solution

Equations (8) - (9) are coupled, non-linear partial differential equations and these equations can not be solved analytically. However, these equations can be solved numerically. One of the most commonly used numerical methods is the finite difference technique which has better stability characteristics and is relatively simple, accurate and efficient. Another essential feature of this technique is that it is based on an iterative procedure and a tri-diagonal matrix manipulation. This method provides satisfactory results but it may fail when applied to problems in which the differential equations are very sensitive to the choice of initial conditions. In all numerical solutions the continuous partial differential equation is replaced with a discrete approximation. In this context the word *discrete* means that the numerical solution is known only at a finite number of points in the physical domain. The number of those points can be selected by the user of the numerical method. In general, increasing the number of points not only increases the resolution but also the accuracy of the numerical solution. The discrete approximation results in a set of algebraic equations that are evaluated (or solved) for the values of the discrete unknowns. The mesh is the set of locations where the discrete solution is computed. These points are called nodes and if one were to draw lines between adjacent nodes in the domain the resulting image would resemble a net or mesh.

When time dependent solutions are important, the Crank-Nicolson scheme has significant advantages. The Crank-Nicolson scheme is not significantly more difficult to implement and it has a temporal truncation error that is $O(\Delta\tau^2)$ as explained by Recktenwald [29]. The Crank-Nicolson scheme is implicit, it is also unconditionally stable [30, 31, 32]. In order to solve the equations (8) and (9) under the initial and boundary conditions (10), an implicit finite difference scheme of Crank-Nicolson's type has been employed. The right hand side of the equations (8) and (9) is approximated with the average of the central difference scheme evaluated at the current and the previous time step. The finite difference equation corresponding to equations

(8) and (9) are as follows:

$$\frac{u_{i,j+1}-u_{i,j}}{\Delta\tau} - S\frac{u_{i+1,j}-u_{i,j}}{\Delta\eta} = \frac{1}{2(\Delta\eta)^2}\left[u_{i-1,j}-2u_{i,j}+u_{i+1,j}+u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}\right] + \frac{Gr}{2}\left(\theta_{i,j+1}+\theta_{i,j}\right) - \frac{1}{2Da}\left(u_{i,j+1}+u_{i,j}\right), \tag{11}$$

$$Pr\left(\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta\tau} - S\frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta\eta}\right) = \frac{1}{2(\Delta\eta)^2}\left[\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j}+\theta_{i-1,j+1}-2\theta_{i,j+1}+\theta_{i+1,j+1}\right] + Pr\,Ec\left(\frac{u_{i+1,j}-u_{i,j}}{\Delta\eta}\right)^2 - \frac{R}{2}\left(\theta_{i,j+1}+\theta_{i,j}\right). \tag{12}$$

The boundary conditions (10) become

$$\begin{aligned} &u_{i,0}=0,\ \theta_{i,0}=0 \ \text{ for all } \ i\neq 0,\\ &u_{0,j}=1,\ \ \theta_{0,j}=1,\\ &u_{N,j}=0,\ \theta_{N,j}=0, \end{aligned} \tag{13}$$

where N corresponds to ∞. Here the suffix i corresponds to y and j corresponds to τ. Also $\Delta\tau=\tau_{j+1}-\tau_j$ and $\Delta\eta=\eta_{i+1}-\eta_i$. Knowing the values of $\theta,\ u$ at a time τ we can evaluate the values at a time $\tau+\Delta\tau$ as follows . We substitute $i=1,2,...,N-1$, in equation (12) which constitute a tri-diagonal system of equations, the system can be solved by Thomas algorithm as discussed in Carnahan et al.[33]. Thus θ is known for all values of η at time τ. Then knowing the values of θ and applying the same procedure with the boundary conditions, we calculate, u from equation (11). This procedure is continued to obtain the solution till desired time τ. The Crank-Nicolson scheme has a truncation error of $O\left(\Delta\tau^2\right)+O\left(\Delta\eta^2\right)$, i.e. the temporal truncation error is significantly small.

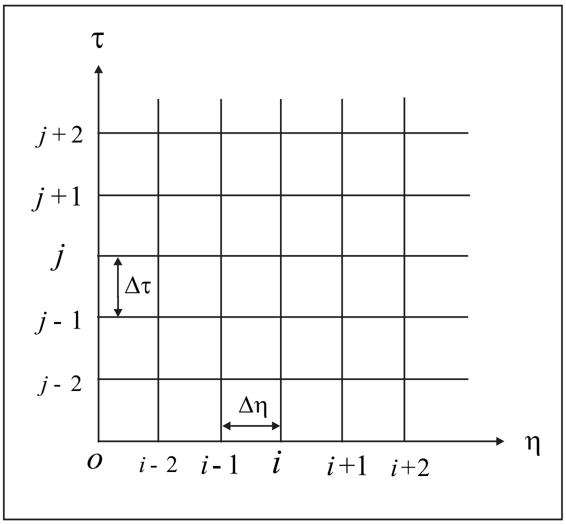

Fig.2: Finite difference grids

The implicit method gives stable solutions and requires matrix inversions which we have done at step forward in time because this problem is an initial-boundary value problem with a finite number of spatial grid points. Though, the corresponding difference equations do not automatically guarantee the convergence of the mesh $\Delta\eta \to 0$. To achieve maximum numerical efficiency, we used the tri-diagonal procedure to solve the two- point conditions governing the main coupled governing equations of momentum and energy. The convergence (consistency) of the process is quite satisfactory and the numerical stability of the method is guaranteed by the implicit nature of the numerical scheme. Hence, the scheme is consistent. The stability and consistency ensure convergence.

4 Results and discussion

We have presented the non-dimensional fluid velocity u and the fluid temperature θ for several values of the radiation parameter R, Darcy number Da, Prandtl number Pr, Grashof number Gr, Eckert number Ec, suction parameter S and time τ in Figs.3-14. It is seen from Fig.3 that the fluid velocity u decreases with an increase in radiation parameter Ra. This implies that the radiation has retarding influence on the velocity field. The radiation parameter arises only in the energy equation in the thermal diffusion term and via coupling of the temperature field with the buoyancy term in the momentum equation, the fluid velocity is indirectly influenced by thermal radiation effect. An increase in Ra clearly reduces the fluid velocity. It is revealed from Fig.4 that the fluid velocity u increases with an increase in Darcy number Da. Fig.5 displays that the fluid velocity u increases with an increase in Prandtl number Pr. Fig.6 shows that the fluid velocity u increases with an increase in Grashof number Gr. Grashof number Gr signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force. As expected, it is observed that there is a rise in the fluid velocity due to the enhancement of thermal buoyancy force. It is due to the fact that an increase of Grashof number has a tendency to increase the thermal effect. This gives rise to an increase in the induced flow. Fig.7 shows that the fluid velocity u increases with an increase in Eckert number Ec. It is seen from Fig.8 that the fluid velocity u decreases with an increase in suction parameter S. It is observed from Fig.9 that the fluid velocity u increases with an increase in time τ. It is illustrated from Fig.10 that the fluid temperature θ increases with an increase in Darcy number Da. Fig.11 display that the fluid temperature θ decreases with an increase in radiation parameter Ra. This is due to the fact that the radiation provides an additional means to diffuse energy. Fig.12 shows that the fluid temperature θ decreases with an increase in Prandtl number Pr. The reason is that smaller values of Prandtl number are equivalent to increasing thermal conductivity and therefore heat is able to diffuse away from the heated surface more rapidly than for higher values of Prandtl number. It is seen from Fig.13 that the fluid temperature θ increases with an increase in Eckert number Ec. Eckert number is the ratio of the kinetic energy of the flow to the boundary layer enthalpy difference. The effect of viscous dissipation on the flow field is to increase the energy, yielding a greater fluid temperature and as a consequence greater buoyancy force. The increase in the buoyancy force due to an increase in the dissipation parameter enhances the temperature. Fig.14 shows that the fluid temperature θ decreases with an increase in suction parameter S. It is seen from Figs.10-14 that the maximum of the temperature occur in the vicinity of the plate and asymptotically approaches to zero in the free stream region.

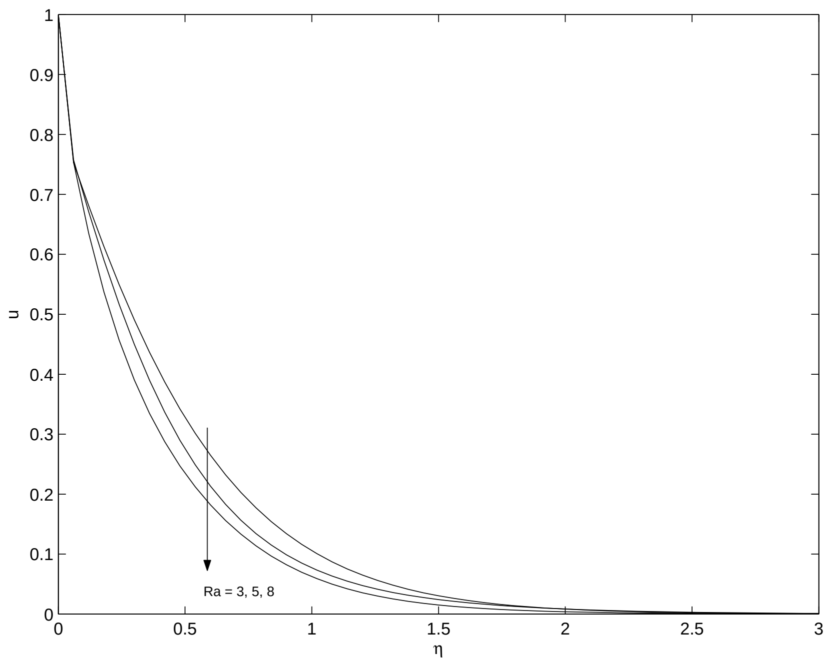

Fig.3: Velocity for Ra when $Da = 0.1$, $Gr = 5$, $Ec = 0.5$, $Pr = 0.25$, $S = 0.5$ and $\tau = 0.2$

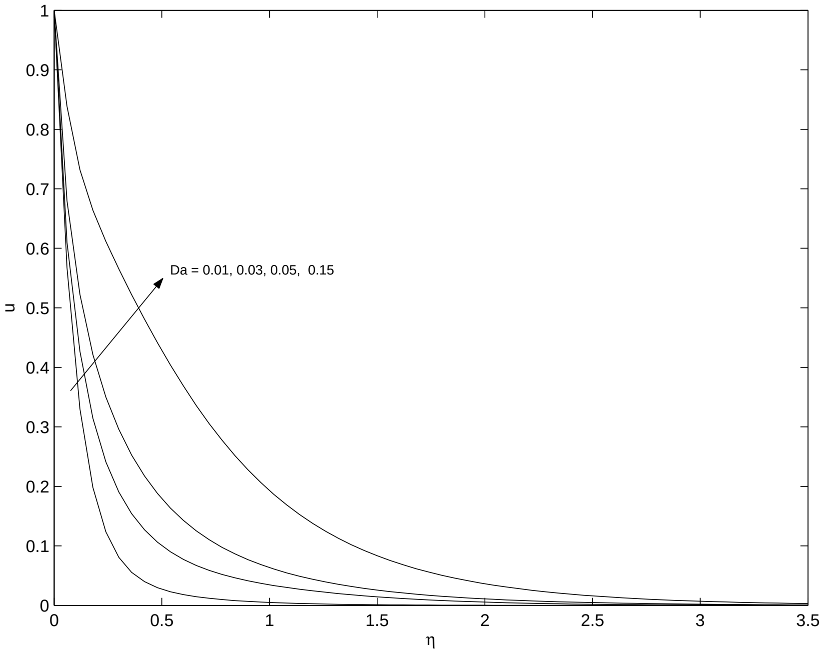

Fig.4: Velocity for Da when $Gr = 5$, $Ra = 2$, $Ec = 0.5$, $Pr = 0.25$, $S = 0.5$ and $\tau = 0.2$

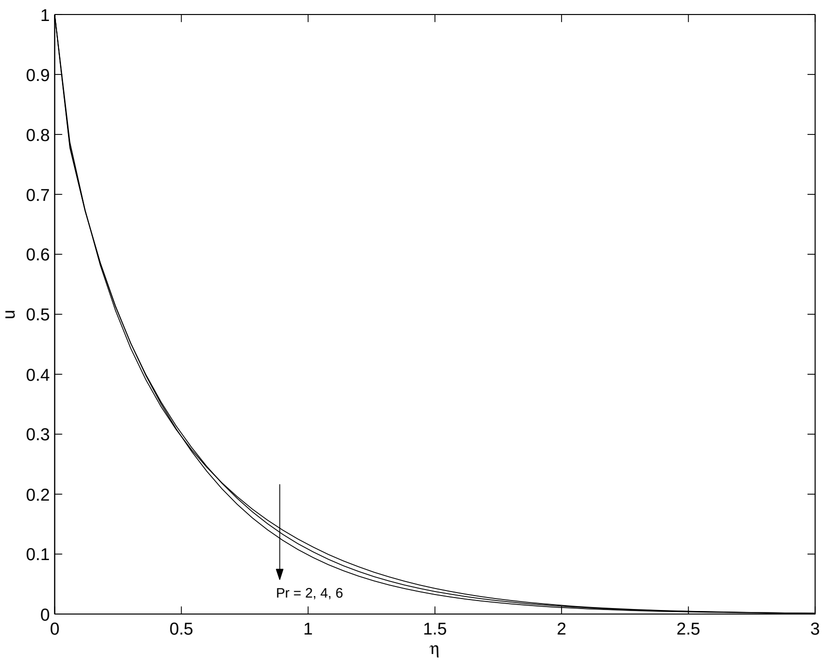

Fig.5: Velocity for Pr when $Da = 0.1$, $Gr = 5$, $Ec = 0.5$, $S = 0.5$ and $\tau = 0.2$

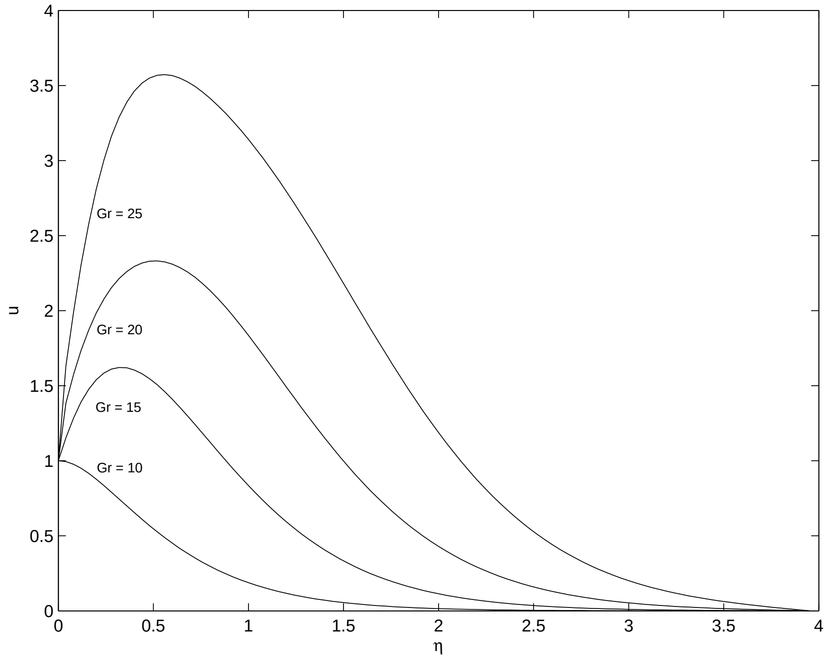

Fig.6: Velocity for Gr when $Da = 0.1$, $Ra = 2$, $Ec = 0.5$, $Pr = 0.25$, $S = 0.5$ and $\tau = 0.2$

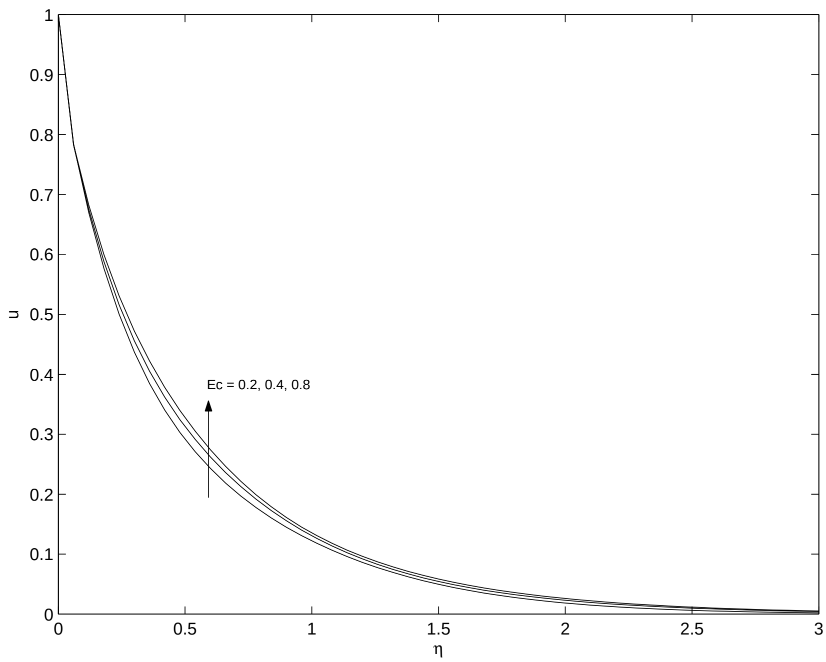

Fig.7: Velocity for Ec when $Da = 0.1$, $Gr = 5$, $Ra = 2$, $Pr = 0.25$, $S = 0.5$ and $\tau = 0.2$

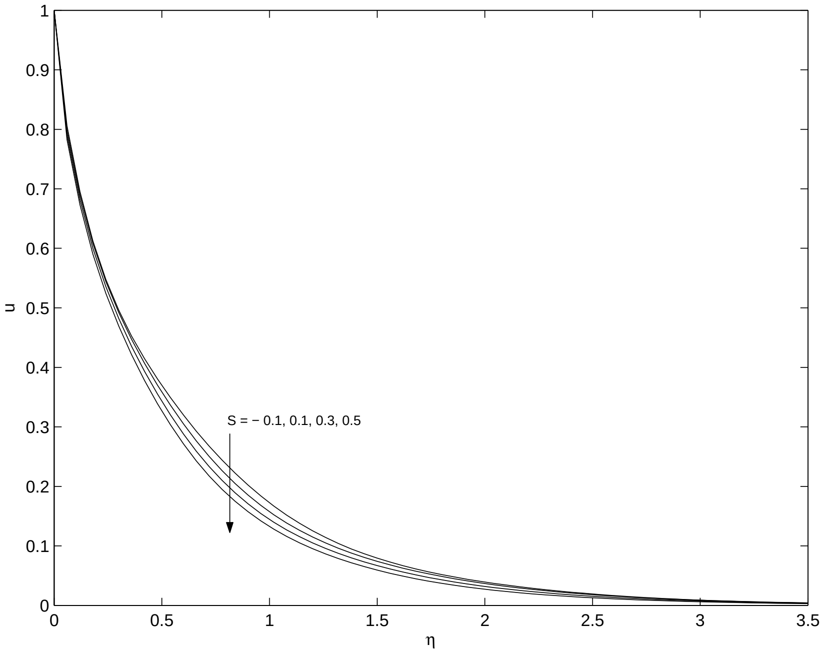

Fig.8: Velocity for S when $Da = 0.1$, $Gr = 5$, $Ra = 2$, $Ec = 0.5$, $Pr = 0.25$ and $\tau = 0.2$

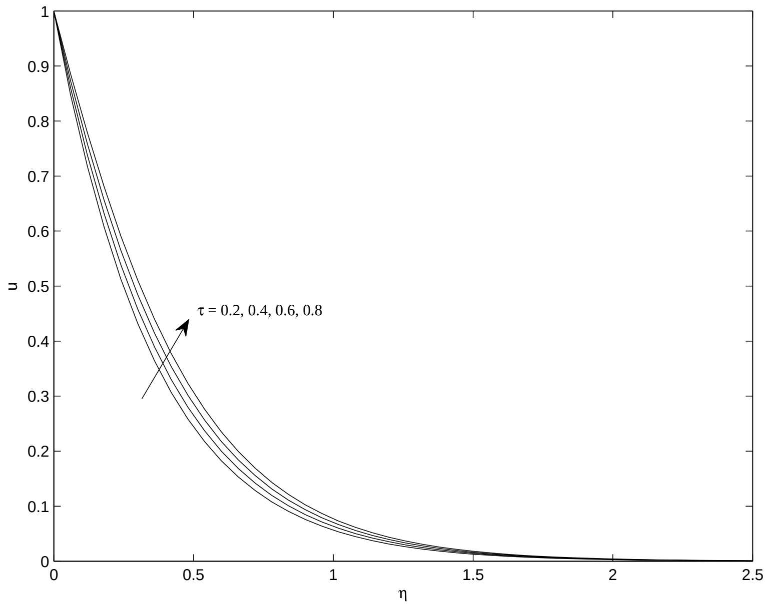

Fig.9: Velocity for time τ when $Da = 0.1$, $Gr = 5$, $Ec = 0.5$, $Pr = 0.25$, $S = 0.5$ and $Ra = 2$

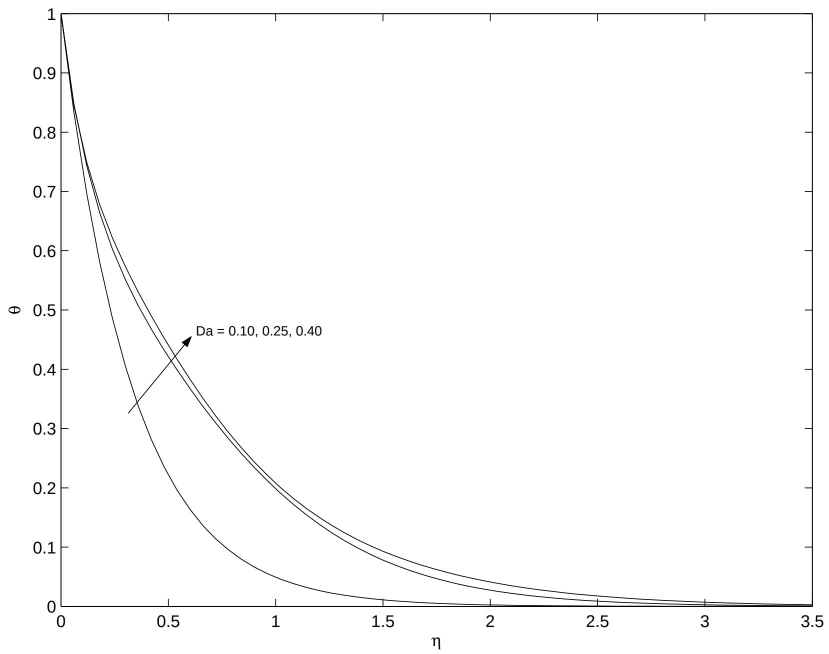

Fig.10: Temperature for Da when $Ra = 2$, $Ec = 0.5$, $Pr = 0.25$, $S = 0.5$ and $\tau = 0.2$

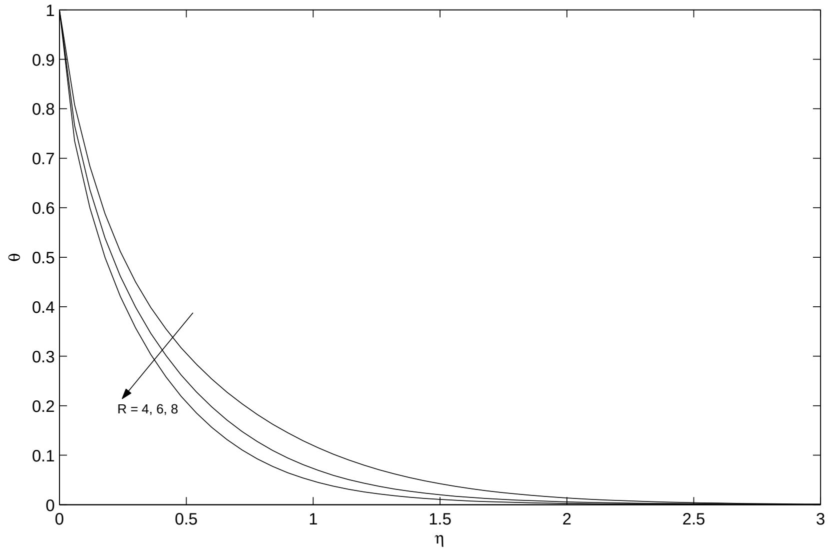

Fig.11: Temperature for Ra when $Da = 0.1$, $Ec = 0.5$, $Pr = 0.25$, $S = 0.5$ and $\tau = 0.2$

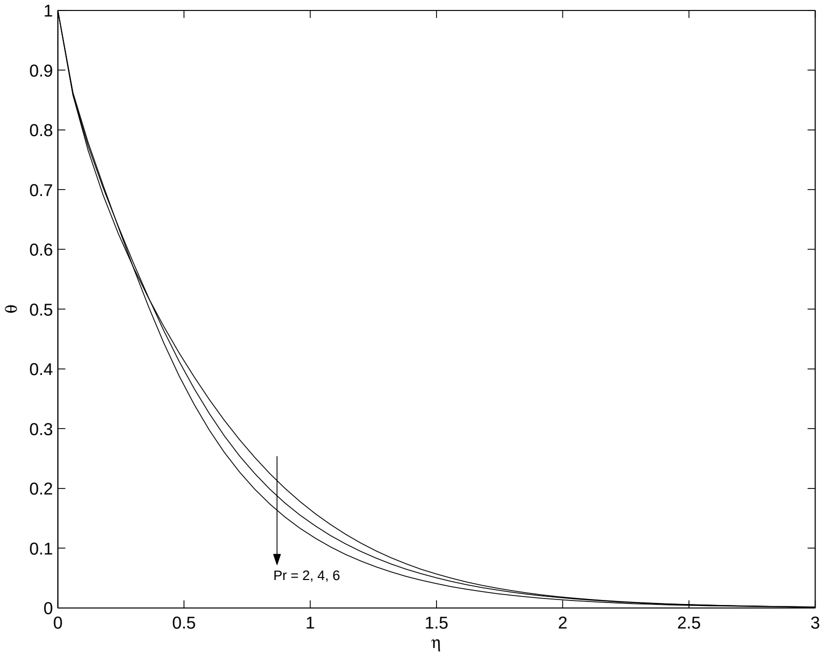

Fig.12: Temperature for Pr when $Da = 0.1$, $Ec = 0.5$, $Ra = 2$, $S = 0.5$ and $\tau = 0.2$

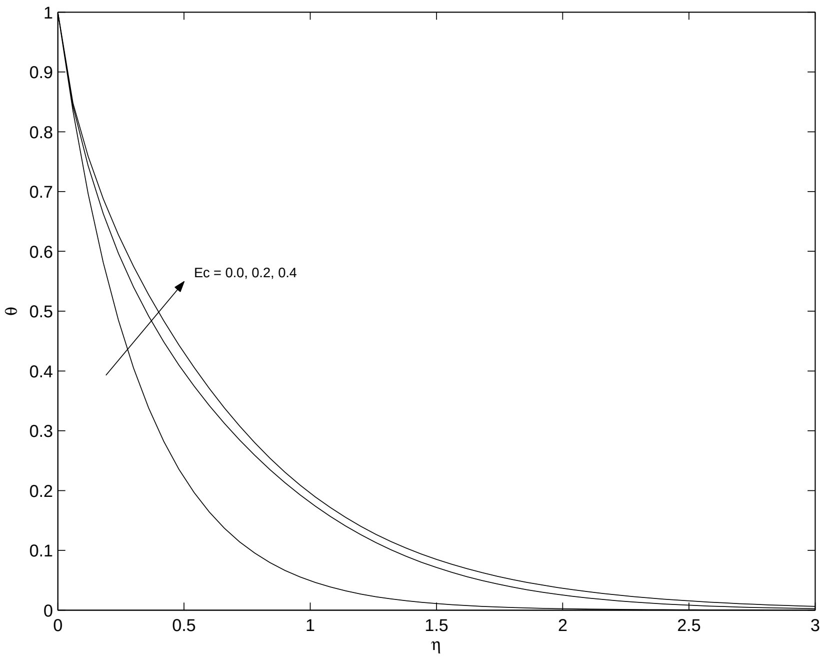

Fig.13: Temperature for Ec when $Da = 0.1$, $Ra = 2$, $Pr = 0.25$, $S = 0.5$ and $\tau = 0.2$

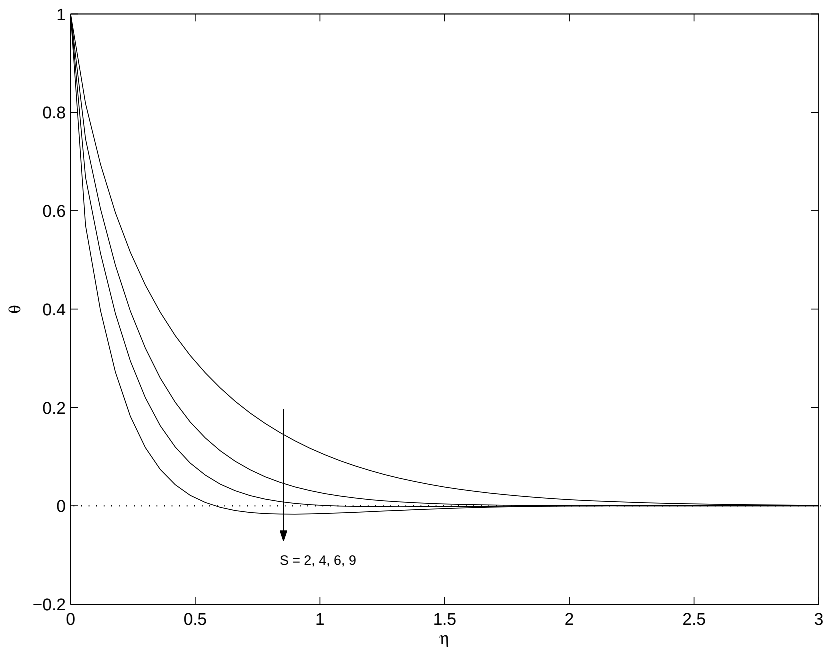

Fig.14: Temperature for S when $Pr = 0.71$, $Da = 0.1$, $Ec = 0.5$ and $Ra = 5$

Numerical values of the rate of heat transfer $-\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}$ at the plate $\eta = 0$ are presented in Tables 1 and 2 for several values of radiation parameter Ra, Prandtl number Pr, time τ, Darcy number Da and Eckert number Ec. It is seen from Table 1 that the rate of heat transfer $-\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}$ at the plate $\eta = 0$ decreases with an increase in either Prandtl numbe Pr or time τ. It is also seen that the rate of heat transfer $-\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}$ at the plate $\eta = 0$ increases with an increase in radiation parameter Ra. Further, it is seen from Table 2 that the rate of heat transfer $-\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}$ decreases with an increase in Darcy number Da while it decreases with an increase in time τ for fixed values of radiation parameter Ra. The positive values of the rate of heat transfer show that the heat is transferred from the medium to the plate.

Table 1. Rate of heat transfer $-\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}$ at the plate $\eta = 0$

	Pr				τ			
Ra	0.25	0.5	0.71	0.85	0.2	0.4	0.6	0.8
2	2.44407	2.26802	2.15944	2.10457	3.56816	3.26816	3.00816	2.56816
4	3.48842	3.19362	3.17673	3.15100	3.64052	3.36210	3.03399	2.65619
6	3.88086	3.81167	3.80541	3.74681	3.71281	3.60352	3.47549	3.32894
8	4.53613	4.37582	4.29178	4.24777	4.32732	4.15019	3.94108	3.70008

Table 2. Rate of heat transfer $-\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}$ at the plate $\eta=0$

	Da				Ec			
R	0.01	0.02	0.03	0.04	0.2	0.4	0.6	0.8
2	3.56816	3.12768	3.06254	3.06111	3.56816	3.23749	3.18157	3.07828
4	3.59562	3.17326	3.17135	3.16223	3.67858	3.28561	3.32644	3.08256
6	3.76693	3.65978	3.66987	3.70648	3.88739	3.88868	3.81513	3.71984
8	4.30862	4.27573	4.26754	4.26310	4.60096	4.37165	4.24554	4.13820

The non-dimensional shear stress τ_x at the plate $\eta=0$ due to the flow is given by

$$\tau_x = \left(\frac{\partial u}{\partial\eta}\right)_{\eta=0}. \tag{14}$$

Numerical values of the non-dimensional shear stress τ_x due to the flow at the plate $\eta=0$ are presented in Figs.15-19 against radiation parameter R for several values of Darcy number Da, Prandtl number Pr, Grashof number Gr, suction parameter S, Eckert number Ec and time τ. Fig.15 shows that the absolute value of the shear stress τ_x increases with an increase in Darcy number Da. On other hand, it is observed that the absolute value of the shear stress τ_x increases with an increase in radiation parameter Ra. Fig.16 shows that the absolute value of the shear stress τ_x increases with an increase in Prandtl number Pr. Physically, this is true because the increase in the Prandtl number is due to increase in the viscosity of the fluid, which makes the fluid thick and hence a decrease in the velocity of the fluid. Fig.17 shows that the absolute value of the shear stress τ_x decreases with an increase in Grashof number Gr. It is illustrated from Fig.18 that the absolute value of the shear stress τ_x increases with an increase in suction parameter S. It is seen from Fig.19 that the absolute value of the shear stress τ_x decreases with an increase in Eckert number Ec. Fig.20 shows that the absolute value of the shear stresses τ_x increases with an increase in time τ.

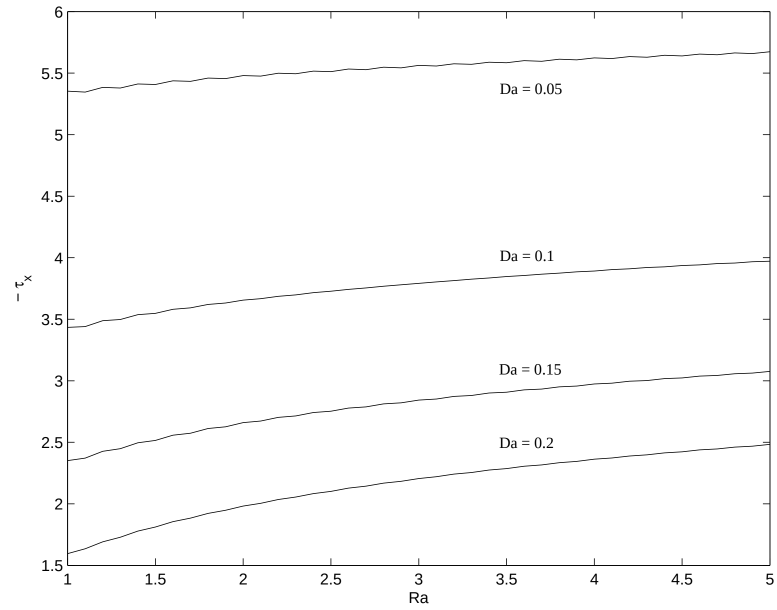

Fig.15: Shear stress τ_x for Da when $Pr = 0.71$, $Gr = 5$, $S = 0.5$, $Ec = 0.5$ and $\tau = 0.2$

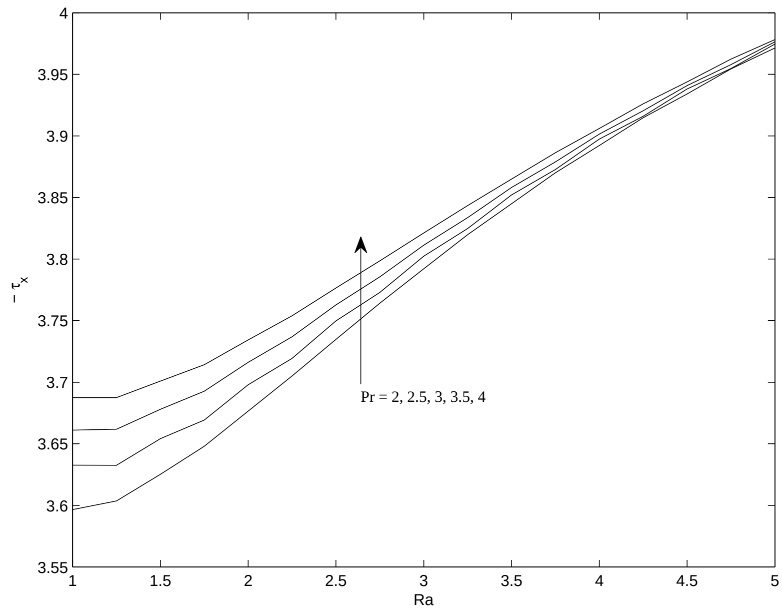

Fig.16: Shear stress τ_x for Pr when $Da = 0.1$, $Gr = 5$, $S = 0.5$, $Ec = 0.5$ and $\tau = 0.2$

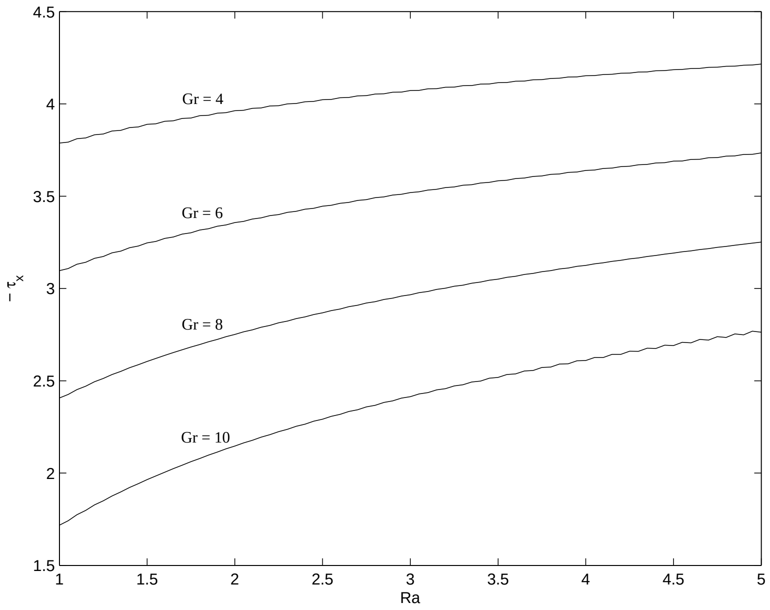

Fig.17: Shear stress τ_x for Gr when $Da = 0.1$, $Pr = 0.71$, $S = 0.5$, $Ec = 0.5$ and $\tau = 0.2$

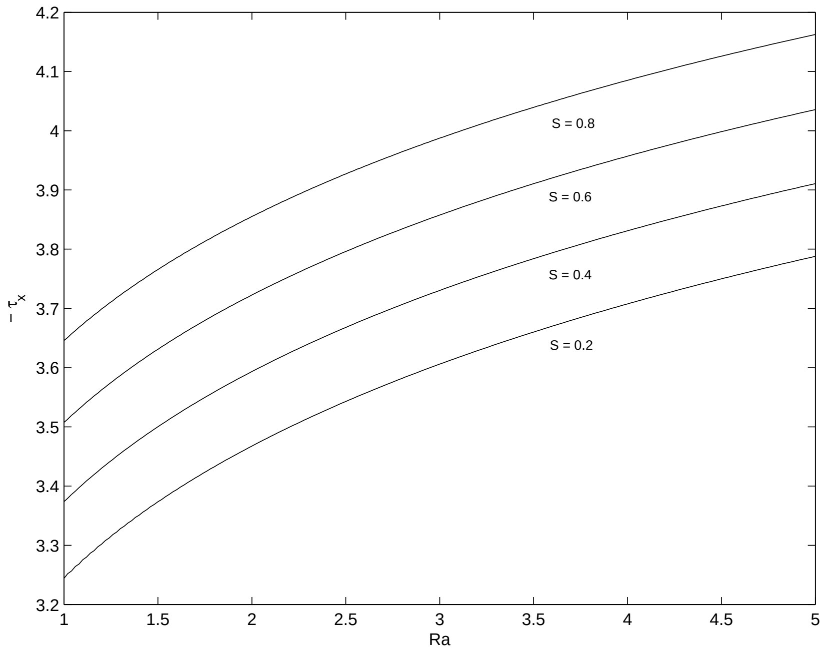

Fig.18: Shear stress τ_x for S when $Da = 0.1$, $Pr = 0.71$, $Gr = 5$, $Ec = 0.5$ and $\tau = 0.2$

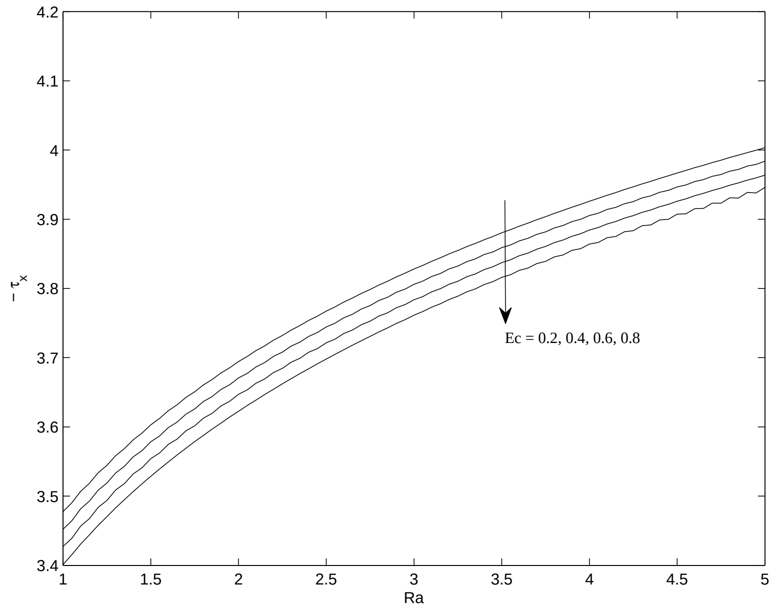

Fig.19: Shear stress τ_x for Ec when $Da = 0.1$, $Pr = 0.71$, $Gr = 5$, $S = 0.5$ and $\tau = 0.2$

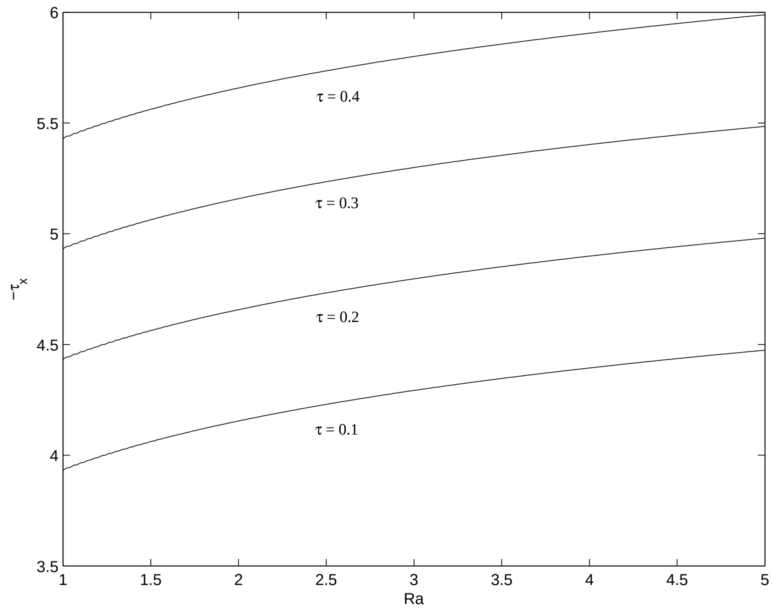

Fig.20: Shear stress τ_x for τ when $Da = 0.1$, $Pr = 0.71$, $Gr = 5$, $S = 0.5$ and $Ec = 0.5$

5 Conclusion

The radiation effects on unsteady free convection flow of a viscous incompressible fluid past a moving vertical plate embedded in porous medium by taking into account viscous dissipation have been studied. It is observed that the radiation has a retarding influence on the fluid velocity. An increase in Darcy number leads to increase the fluid velocity. The fluid velocity and temperature raise due to viscous dissipation. The presence of suction falls the fluid velocity and temperature. Further, it is found that the absolute value of the shear stress τ_x at the plate $(\eta = 0)$ increases with an increase in either Ra or τ. The rate of heat transfer $-\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}$ at the plate $(\eta = 0)$ increases with an increase in Ra. It is also found that the rate of heat transfer falls with increasing Eckert number Ec.

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