

Aligned Magnetic Field and Chemical Reaction Effects on Flow past a Vertical Oscillating Plate through Porous Medium

N.Sandeep¹, Dr.V.Sugunamma²

¹Assistant Professor, Department of mathematics, School of Engineering and Technology, Jain University, Bangalore, India

²Associate Professor, Department of mathematics, S.V.U College of Sciences, S.V.University, Tirupati, India

Corresponding author: N.Sandeep, Department of mathematics, School of Engineering and Technology, Jain University, Bangalore, India

ABSTRACT: Present paper deals with the first order chemical reaction effects on unsteady free convective flow of a viscous incompressible flow past an infinite isothermal vertical oscillating plate with mass transfer in the presence of aligned magnetic field and heat generation /absorption. Exact solution for the dimensionless governing equations has been obtained by the Laplace transform method, when the plate is oscillating harmonically in its own plane. The effects of velocity, temperature and concentration are studied for different parameters.

Key words: Chemical Reaction, MHD, Heat source, Convection, inclination.

1 INTRODUCTION

The experimental and theoretical works on MHD flow with thermal diffusion and chemical reaction have been done extensively in various areas i.e. sustain plasma confinement for controlled thermo nuclear fusion, liquid metal cooling of nuclear reactions and electromagnetic casting of metals. Magneto convection plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Important applications in the study of geological formations, in exploration and thermal recovery of oil and in the

assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites.

Seddeek et al [1] examined the effect of chemical reaction and variable viscosity on hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media he has been analysed in the presence of radiation and magnetic field. Chambre and Yang [2] have worked on thermal diffusion of a chemically reactive species in a laminar boundary layer flow. Muthucumaraswamy R and P. Ganesan [3] studied the chemical reaction on the flow past an impulsively started vertical plate with uniform heat and mass flux. The same type of problem with inclusion of constant wall suction was studied by Makinde.O.D. and P. Sibanda [4], Chen [5] discussed heat and mass transfer in MHD flow by natural convection with variable wall temperature and concentration. Devi and Kandsamy [6] discussed the chemical reaction, heat and mass transfer over an accelerating surface with heat source and thermal stratification in the presence of suction and injection. A numerical study of the laminar mixed free forced convection of non-Newtonian power law fluid with mass transfer presented by Eldabe et al. [7]. The influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces was studied by Postelnicu [8] in presence of porous media and Soret and Dufour effects. Sattar [9] analysed the effect of free and forced convection boundary layer flow through a porous medium with large suction.

Ganesan and Palani [10] discussed numerical Crank-Nicholson difference solutions for transient free convection flow from an impulsively started semi-infinite inclined surface. They showed that the time needed to attain the steady state decreases with an increase in the inclination angle to the horizontal. R. Kandasamy and S.P. Anjalidevi [11] investigated the effect of chemical reaction of the flow over a wedge. Magnetic field effects on electrically conducting free

convection heat and mass transfer from an inclined plate with heat generation/absorption was studied by Chamkha et al. [12]. Yih [13] discussed suction/blowing effects on mixed convection about an inclined surfaces in porous media showing numerically that local Nusselt number and the local Sherwood number both increase with suction and decrease with blowing. The effects of transversely applied magnetic field on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar et al [14]. The dimensionless governing equations were solved using Laplace transform technique. Keeping this applications in view

In this paper we analyze the first order chemical reaction effects on unsteady free convective flow of a viscous incompressible flow past an infinite isothermal vertical oscillating plate with mass transfer in the presence inclined magnetic field and heat generation /absorption is considered. Here the plate temperature is raised to T_w and the concentration level near the plate is also raised to C_w^* . An exact solution for the dimensionless governing equations has been obtained by the Laplace transform method, when the plate is oscillating harmonically in its own plane. The effects of velocity, temperature and concentration are studied for different parameters like magnetic field parameter, phase angle, Schmidt number, chemical reaction parameter, thermal Grashof number, heat source parameter, inclined magnetic field angle, mass Grashof number and time.

2 MATHEMATICAL FORMULATION

Here we consider an unsteady flow of a viscous incompressible fluid through porous medium which is infinitely at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C_∞^* . Here the x-axis is taken along the plate in the vertically upward direction and y-axis is taken normal to the plate. Initially it is assumed

that the plate and the fluid are of the same temperature and concentration. At time $t^* > 0$ the plate starts oscillating in its own plane with frequency ω^* and the temperature of the plate is raised to T_w and the concentration level near the plate is also raised to C_w^* . Here the plate is subjected to transverse magnetic field. The fluid considered here is absorbing/emitting heat but a non scattering medium. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. Then by usual Boussinesq approximation, an unsteady flow is governed by the following equations

$$\frac{\partial u}{\partial t^*} = g\beta(T - T_\infty) + g\beta'(C^* - C_\infty^*) + \nu \frac{\partial^2 u}{\partial y^2} - \nu \frac{u}{k'} - M'u \sin^2 \alpha \quad (1)$$

$$\rho C_p \frac{\partial T}{\partial t^*} = k \frac{\partial^2 T}{\partial y^2} - Q_0(T - T_\infty) \quad (2)$$

$$\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial y^2} - k_r(C^* - C_\infty^*) \quad (3)$$

With the following initial and boundary conditions

$$t^* \leq 0 : u = 0, T = T_\infty, C^* = C_\infty^* \quad \text{for all } y$$

$$t^* > 0 : u = u_0 \cos \omega^* t^*, T = T_w, C^* = C_w^* \quad \text{at } y = 0$$

$$u = 0, T \rightarrow T_\infty, C^* \rightarrow C_\infty^* \quad \text{as } y \rightarrow \infty \quad (4)$$

On introducing the following non dimensional quantities

$$U = \frac{u}{u_0}, t = \frac{t^* u_0^2}{\nu}, Y = \frac{y u_0}{\nu}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, M' = \frac{\sigma B_0^2}{\rho}$$

$$C = \frac{C_w^* - C_\infty^*}{C_w^* - C_\infty^*}, Gr = \frac{g \beta v (T_w - T_\infty)}{u_0^3}, Gc = \frac{g \beta' v (C_w^* - C_\infty^*)}{u_0^3}$$

$$\omega = \frac{\omega^* v}{u_0^2}, Q = \frac{Q_0 v}{\rho C_p u_0^2}, Pr = \frac{\mu C_p}{K}, k_0 = \frac{k' v^2}{u_0^2}$$

$$Sc = \frac{v}{D}, M = \frac{\sigma B_0^2 v}{\rho u_0^2}, k = \frac{v k_r}{u_0^2} \tag{5}$$

Equations (1) to (4) reduces to

$$\frac{\partial u}{\partial t} = Gr \theta + Gc C + \frac{\partial^2 u}{\partial Y^2} - \left(M \sin^2 \alpha + \frac{1}{k_0} \right) U \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} - Q \theta \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - k C \tag{8}$$

The initial and boundary conditions are in non-dimensional form are

$$t \leq 0 : U = 0, \theta = 0, C = 0 \quad \forall Y$$

$$t > 0 : U = \cos \omega t, \theta = 1, C = 1 \quad \text{at } Y = 0$$

$$U = 0, \theta \rightarrow \infty, C \rightarrow 0 \quad \text{as } Y \rightarrow \infty \tag{9}$$

3. SOLUTION OF THE PROBLEM

The solutions of equations (6) to (8) are obtained for hydrodynamic flow field in the presence of first order chemical reaction. The equation subject to the boundary conditions are solved by the usual Laplace transform technique and the solutions are derived as follows

$$\theta = \frac{1}{2} \left[\exp(2\xi \sqrt{Qt}) \operatorname{erfc}(\xi \sqrt{Pr} + \sqrt{At}) + \exp(-2\xi \sqrt{Qt}) \operatorname{erfc}(\xi \sqrt{Pr} - \sqrt{At}) \right] \tag{10}$$

$$C = \frac{1}{2} \left[\exp(2\xi\sqrt{ktSc}) \operatorname{erfc}(\xi\sqrt{Sc} + \sqrt{kt}) + \exp(-2\xi\sqrt{ktSc}) \operatorname{erfc}(\xi\sqrt{Sc} - \sqrt{kt}) \right] \quad (11)$$

$$U = \frac{\exp(i\omega t)}{4} \left[\exp(2\xi\sqrt{(M \sin^2 \alpha + \frac{1}{k_0} + i\omega)t}) \operatorname{erfc}(\xi + \sqrt{(M \sin^2 \alpha + \frac{1}{k_0} + i\omega)t}) \right. \\ \left. + \exp(-2\xi\sqrt{(M \sin^2 \alpha + \frac{1}{k_0} + i\omega)t}) \operatorname{erfc}(\xi - \sqrt{(M \sin^2 \alpha + \frac{1}{k_0} + i\omega)t}) \right] \\ + \frac{\exp(-i\omega t)}{4} \left[\exp(2\xi\sqrt{(M \sin^2 \alpha + \frac{1}{k_0} - i\omega)t}) \operatorname{erfc}(\xi + \sqrt{(M \sin^2 \alpha + \frac{1}{k_0} - i\omega)t}) \right. \\ \left. + \exp(-2\xi\sqrt{(M \sin^2 \alpha + \frac{1}{k_0} - i\omega)t}) \operatorname{erfc}(\xi - \sqrt{(M \sin^2 \alpha + \frac{1}{k_0} - i\omega)t}) \right] \\ + (D + E) \left[\exp(2\xi\sqrt{(M \sin^2 \alpha + \frac{1}{k_0})t}) \operatorname{erfc}(\xi + \sqrt{(M \sin^2 \alpha + \frac{1}{k_0})t}) \right. \\ \left. + \exp(-2\xi\sqrt{(M \sin^2 \alpha + \frac{1}{k_0})t}) \operatorname{erfc}(\xi - \sqrt{(M \sin^2 \alpha + \frac{1}{k_0})t}) \right] \\ - D \exp(At) \left[\exp(2\xi\sqrt{(M \sin^2 \alpha + \frac{1}{k_0} + B)t}) \operatorname{erfc}(\xi + \sqrt{(M \sin^2 \alpha + \frac{1}{k_0} + B)t}) \right. \\ \left. + \exp(-2\xi\sqrt{(M \sin^2 \alpha + \frac{1}{k_0} + B)t}) \operatorname{erfc}(\xi - \sqrt{(M \sin^2 \alpha + \frac{1}{k_0} + B)t}) \right] \\ - E \exp(Bt) \left[\exp(2\xi\sqrt{(M \sin^2 \alpha + \frac{1}{k_0} + F)t}) \operatorname{erfc}(\xi + \sqrt{(M \sin^2 \alpha + \frac{1}{k_0} + F)t}) \right. \\ \left. + \exp(-2\xi\sqrt{(M \sin^2 \alpha + \frac{1}{k_0} + F)t}) \operatorname{erfc}(\xi - \sqrt{(M \sin^2 \alpha + \frac{1}{k_0} + F)t}) \right] \\ - D \left[\exp(2\xi\sqrt{Qt}) \operatorname{erfc}(\xi\sqrt{Pr} + \sqrt{At}) + \exp(-2\xi\sqrt{Qt}) \operatorname{erfc}(\xi\sqrt{Pr} - \sqrt{At}) \right] \\ - D \exp(At) \left[\exp(2\xi\sqrt{Pr(A+B)t}) \operatorname{erfc}(\xi\sqrt{Pr} + \sqrt{(A+B)t}) \right. \\ \left. + \exp(-2\xi\sqrt{Pr(A+B)t}) \operatorname{erfc}(\xi\sqrt{Pr} - \sqrt{(A+B)t}) \right]$$

$$\begin{aligned}
 & -\frac{D}{2} \left[\exp(2\xi \sqrt{ktSc}) \operatorname{erfc}(\xi \sqrt{Sc} + \sqrt{kt}) + \exp(-2\xi \sqrt{ktSc}) \operatorname{erfc}(\xi \sqrt{Sc} - \sqrt{kt}) \right] \\
 & + E \exp(Bt) \left[\begin{aligned} & \exp(2\xi \sqrt{Sc(k+F)t}) \operatorname{erfc}(\xi \sqrt{Sc} + \sqrt{(k+F)t}) \\ & + \exp(-2\xi \sqrt{Sc(k+F)t}) \operatorname{erfc}(\xi \sqrt{Sc} - \sqrt{(k+F)t}) \end{aligned} \right]
 \end{aligned}
 \tag{12}$$

Where

$$A = \frac{Q}{Pr}, \quad B = \frac{M + \frac{1}{k_0} - Q}{Pr - 1}, \quad F = \frac{M + \frac{1}{k_0} - kSc}{Sc - 1}$$

$$D = \frac{Gr}{2B(1 - Pr)} \quad \text{and} \quad E = \frac{Gc}{2F(1 - Sc)}$$

Where $\xi = \frac{Y}{2\sqrt{t}}$ and erfc is called complementary error

function

4 RESULTS AND DISCUSSION

Some numerical calculations have been carried out for the non-dimensional velocity (U), the temperature (θ), the concentration (C). The effects of material parameters such as the Prandtl number (Pr), the Schmidt number (Sc), the magnetic parameter (M), the porosity parameter (k_0), the chemical reaction parameter (k), the heat source parameter (Q), the Grashof number (Gr), the phase angle (ωt), the angle of inclination (α) and time (t) have been observed. The numerical calculations of these results are presented graphically in Figs. 1-

15. During the course of numerical calculations of the velocity, the temperature, and the concentration, the values of the Prandtl number are chosen for air ($Pr=0.71$), water ($Pr =7.0$) and water at $4^{\circ}C$ ($Pr=11.40$), the phase angle ($\omega t = \frac{\pi}{2}$), Grashof number ($Gr = Gc = 5$), inclined angle ($\alpha = \frac{\pi}{2}$).

Fig.1 illustrates the effect of Schmidt number on the velocity profiles from this illustration it is observed that as the Schmidt number increases. the fluid velocity also increases. The influence of Grashof number on the velocity field is illustrated in fig. 2 for a fixed porosity on the bounding surfaces and as the Grashof number increases, the velocity is increases. Fig. 3 shows the effect of porosity parameter on the velocity profiles. It is clearly illustrated that as the increase of porosity causes the decrease of velocity of the fluid. Fig. 4 shows the influence of inclination angle on the velocity profiles. As the angle of inclination increases causes the decrease of velocity. Fig. 5 shows the effect of chemical reaction parameter on dimensionless velocity .It is observed that the velocity increases with increasing chemical reaction parameter. Fig.6 illustrates the effect of phase angle on the velocity profiles from these illustration it is observed that as the phase angle increases the fluid velocity decreases. Fig.7 shows that the rapid increase of velocity at the plate, attains a maximum near the plate, and decreases to the free stream value away from the plate by decrease of Prandtl number. From fig.8 it is clear that the velocity profiles increases with decrease of magnetic field M when $Gr > 0$, at lower magnetic field effect velocity profiles attains its maximum value near the plate. And in case of $Gr < 0$ it is reversed. Fig. 9 shows effect of time on the velocity profiles, it is observed that the increase of time casus the decrease of velocity. Fig.10 shows the effect of heat source parameter on velocity profiles .From these it is noticed that velocity of the fluid increases with increase in heat source parameter.

Figs.11 and 12 shows the effects of prandtl number and heat source parameter on temperature profiles it is observed that increasing the prandtl number and heat source parameter casus the decrease of temperature profiles. From fig. 13 we observe that the increase of chemical reaction parameter causes the gradual decrease in concentration profiles.Fig.14 shows that the effect of Schmidt number on concentration profiles here we observe that the increase of Schmidt number causes the decrease in concentration profiles. In the similar manner from fig.15 we observe that increase of time casus the decrease of concentration profiles.

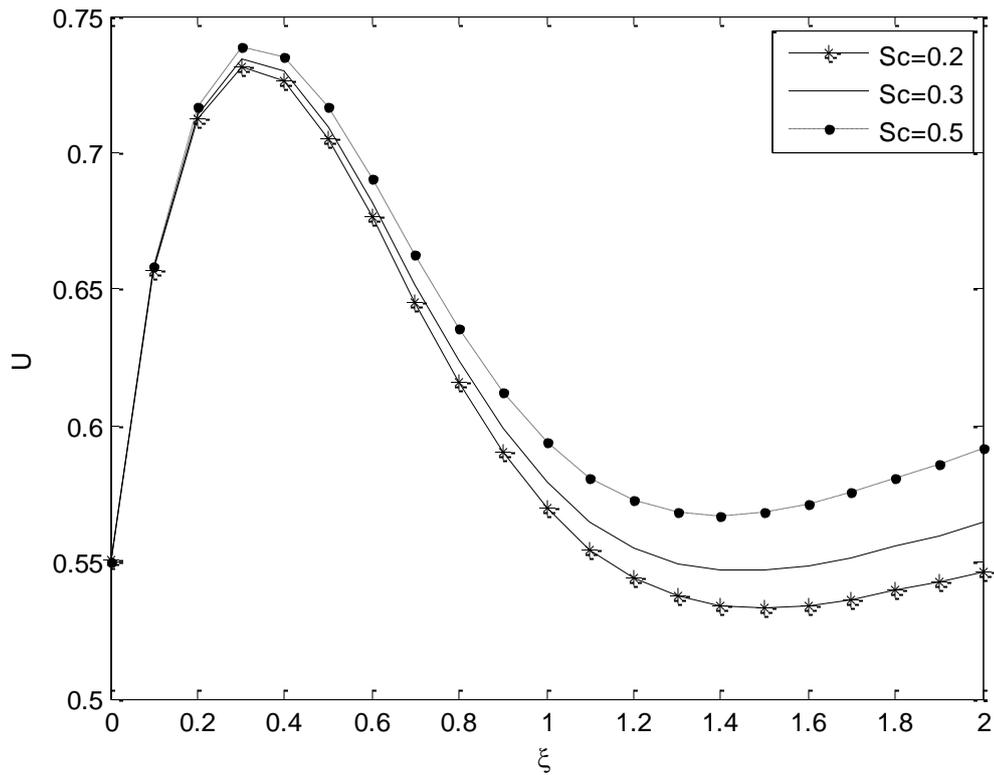


Fig 1.Variation of velocity for different values of Schmidt number

When $K=5, Q=3, M=1, t=0.001, Ko=0.01.$

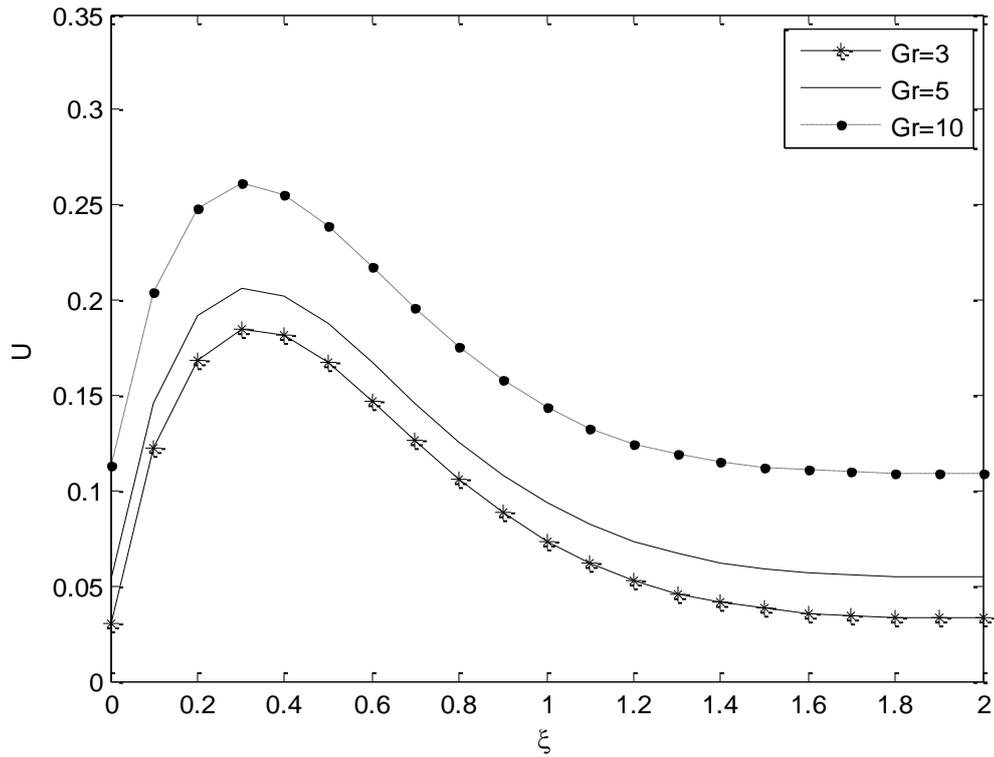


Fig 2.Variation of velocity for different values of Grashof number

When $Sc=0.2;K=5,Q=3,M=1,t=0.001,Ko=0.01$.

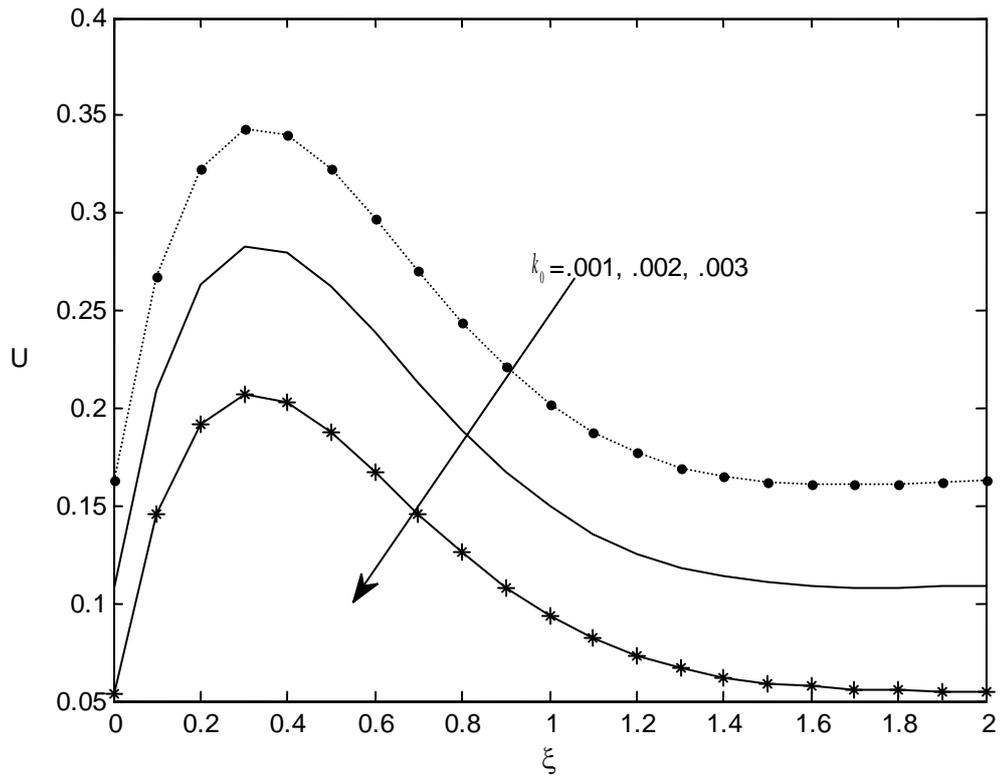


Fig 3. Variation of velocity for different values of porosity

When $Sc=0.2, K=5, Q =3, M=1, t=0.001$.

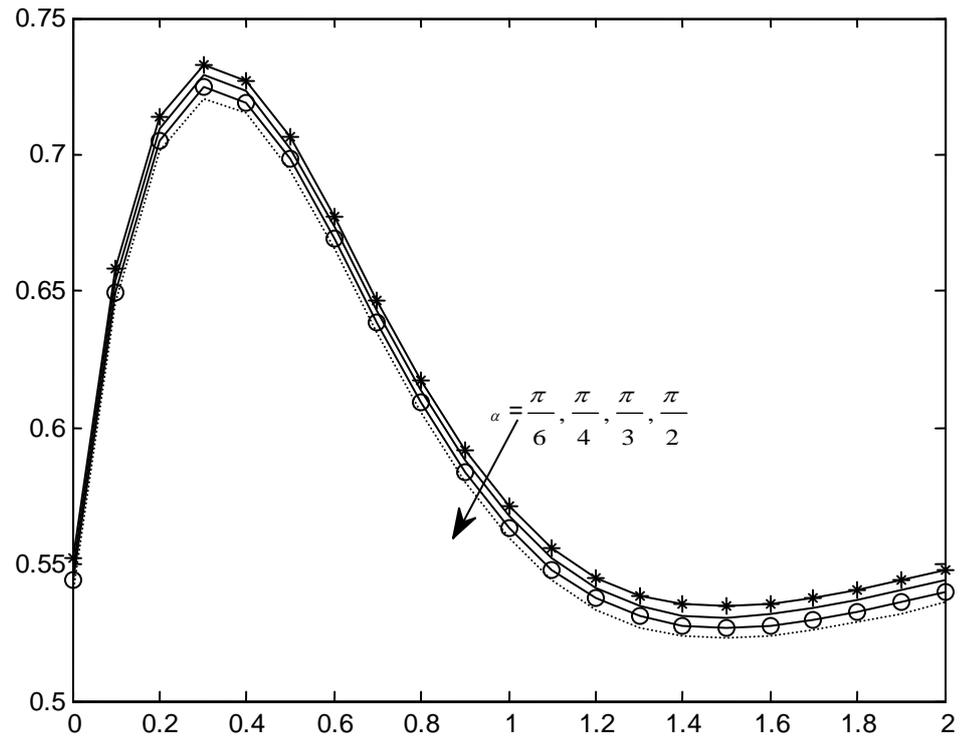


Fig 4. Variation of velocity for different values of inclined angle

When $Sc=0.2$, $K=5$, $Q=3$, $M=3$, $t=0.001$, $Ko=0.01$.

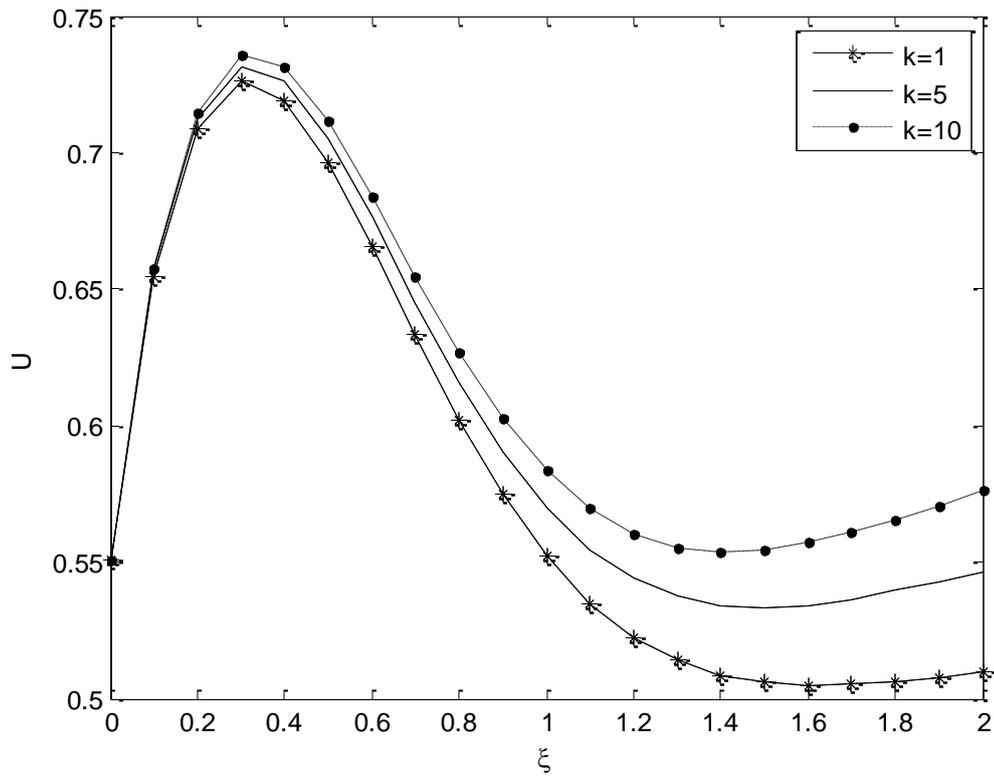


Fig5. Variation of velocity for different values of chemical reaction parameter

When $Sc=0.2, Q=3, M=1, t=0.001, Ko=0.01$.

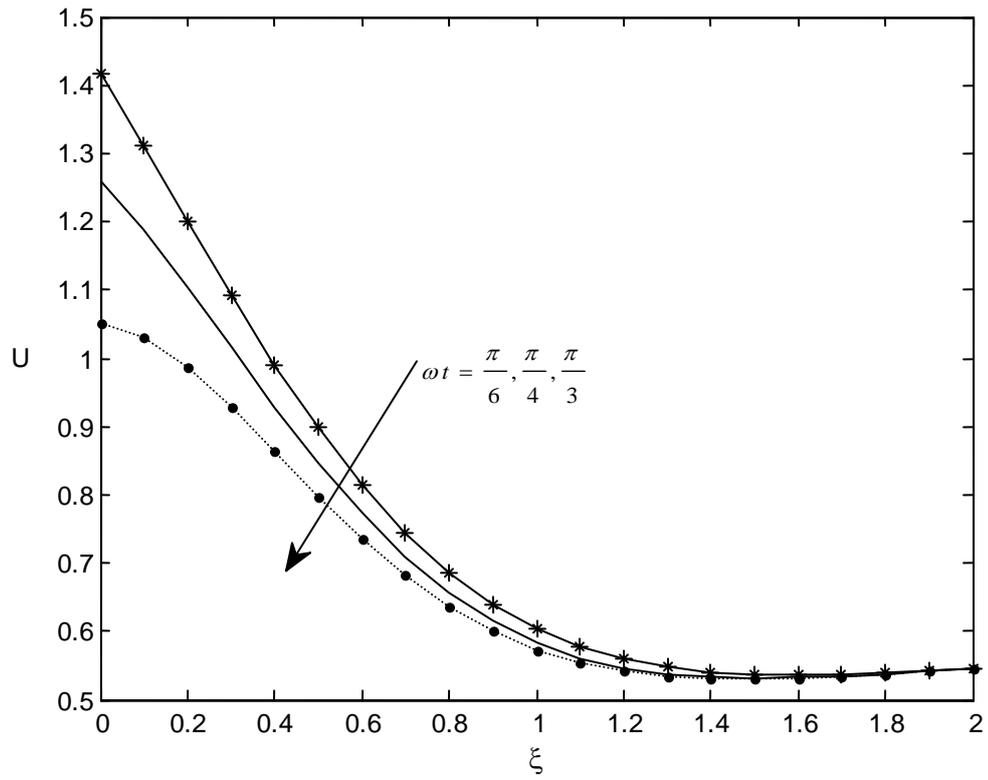


Fig 6. Variation of velocity for different values of phase angle

When $Sc=0.2$, $k=5$, $Q=3$, $M=1$, $t=0.001$, $Ko=0.01$.

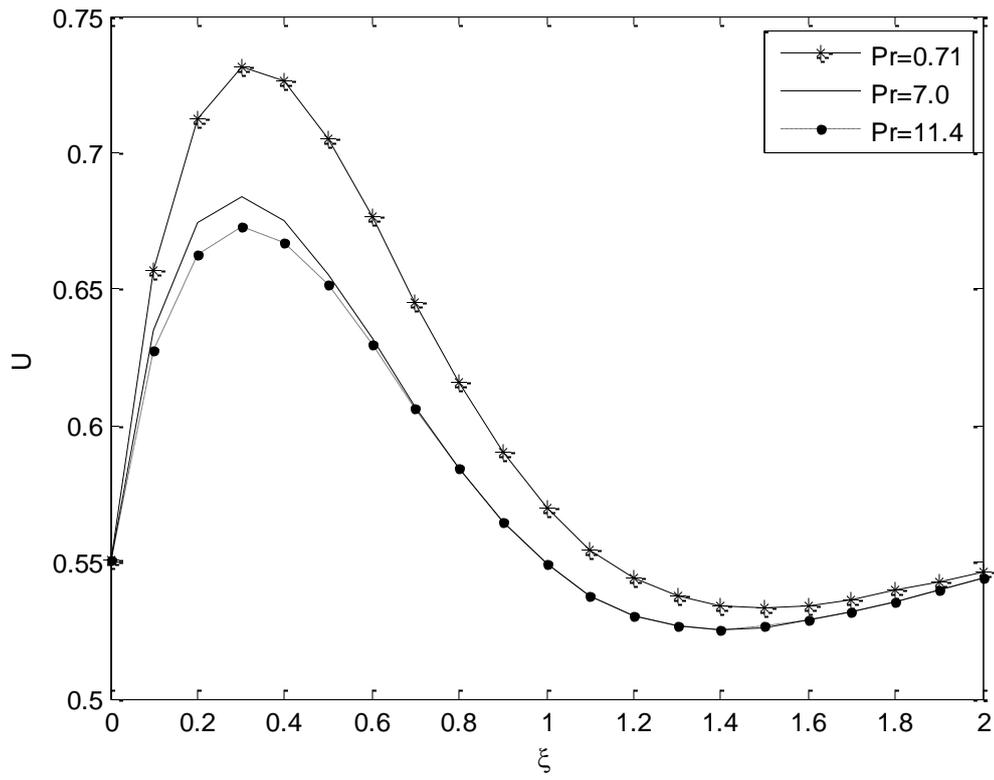


Fig 7. Variation of velocity for different values of prandtl number

When $Sc=0.2, k=5, Q=3, M=1, t=0.001, Ko=0.01$.

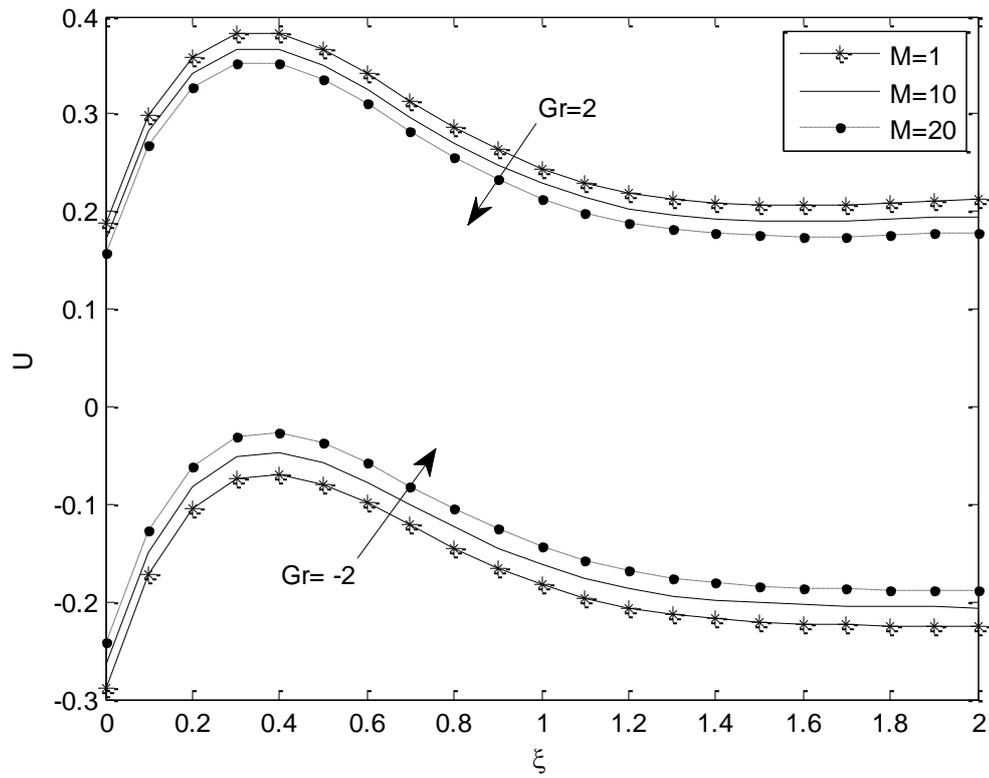


Fig 8. Variation of velocity for different values of Magnetic field

When $Sc=0.2$, $k=5$, $Q=2$, $t=0.001$, $Ko=0.01$.

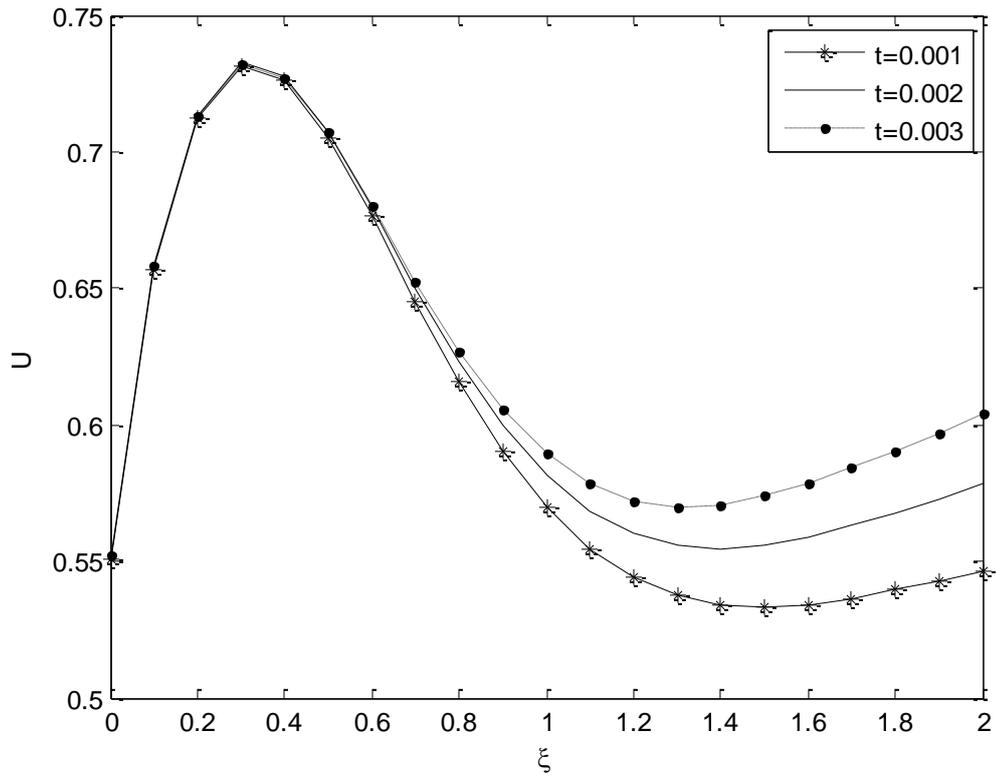


Fig 9. Variation of velocity for different values of time

When $Sc=0.2, k=5, Q=3, M=1, t=0.001, Ko=0.01$.

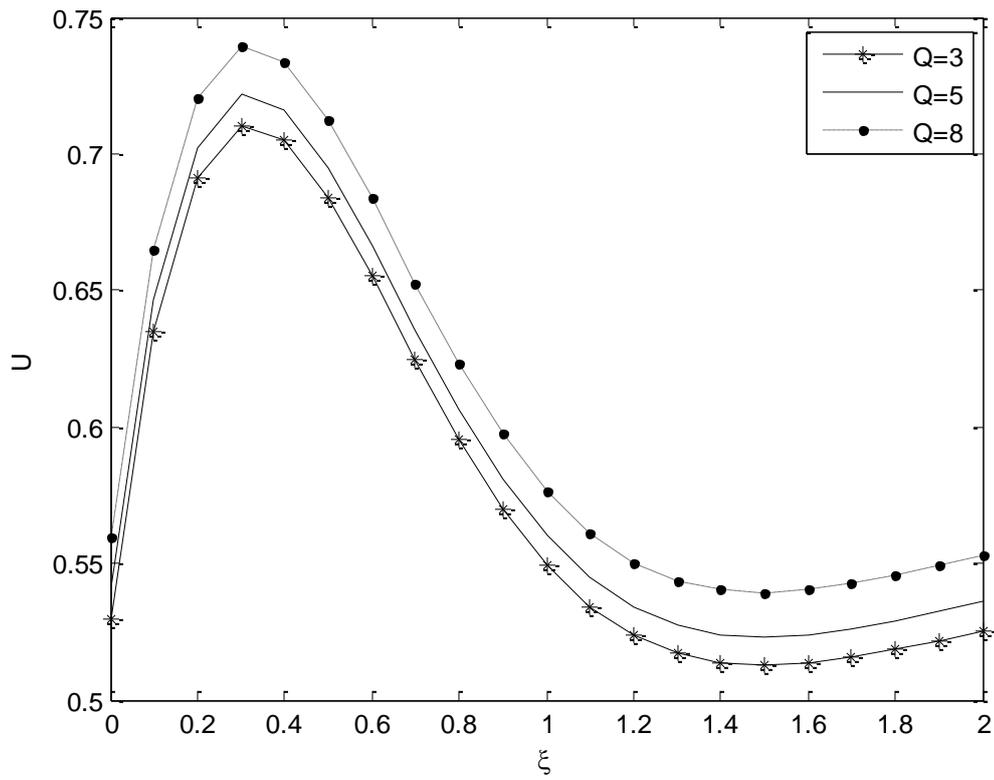


Fig 10. Variation of velocity for different values of heat source parameter

When $Sc=0.2$, $k=5$, $M=5$, $t=0.001$, $Ko=0.01$.

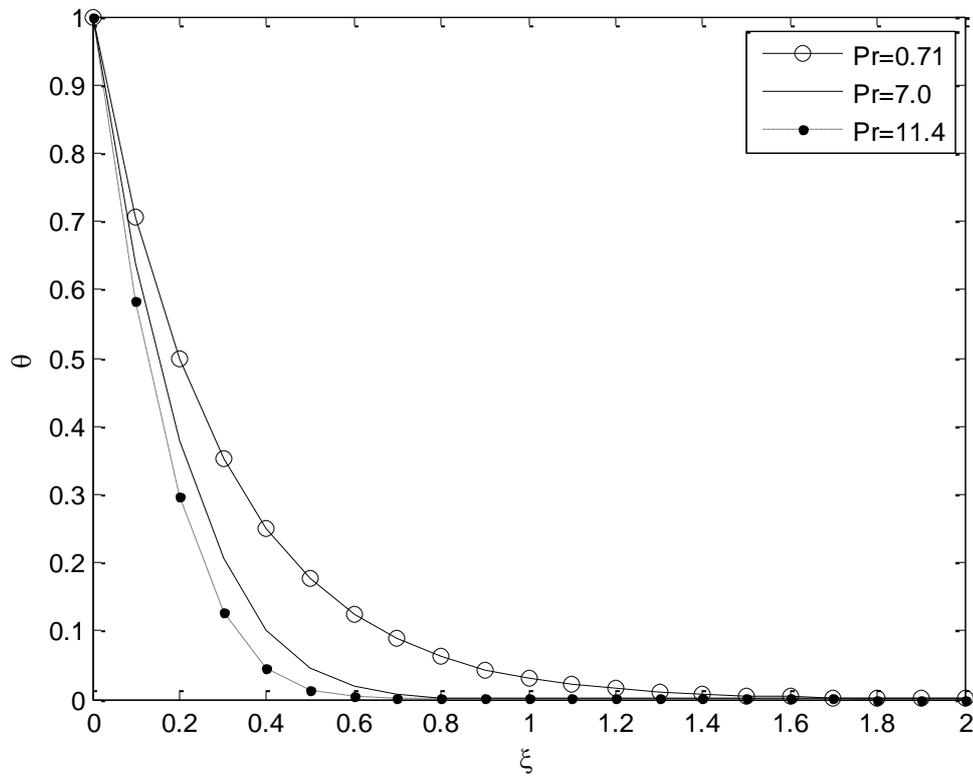


Fig 11. Variation of temperature for different values of Prandtl number

When $Sc=0.4$, $k=5$, $Q=2$, $M=1$, $t=0.5$, $Ko=0.1$.

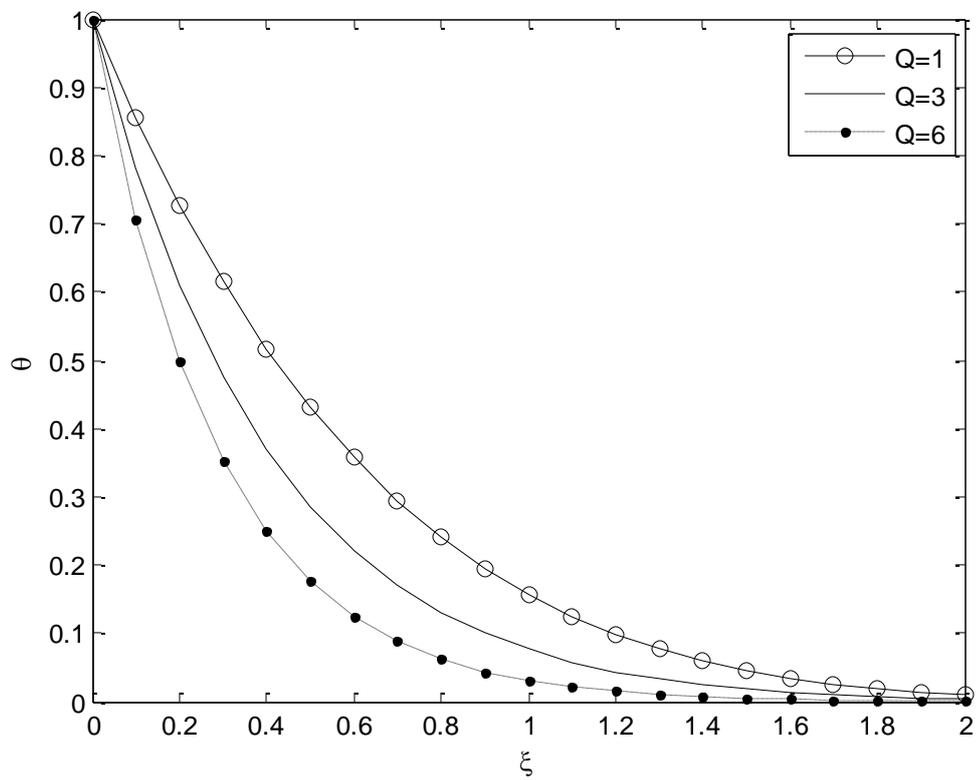
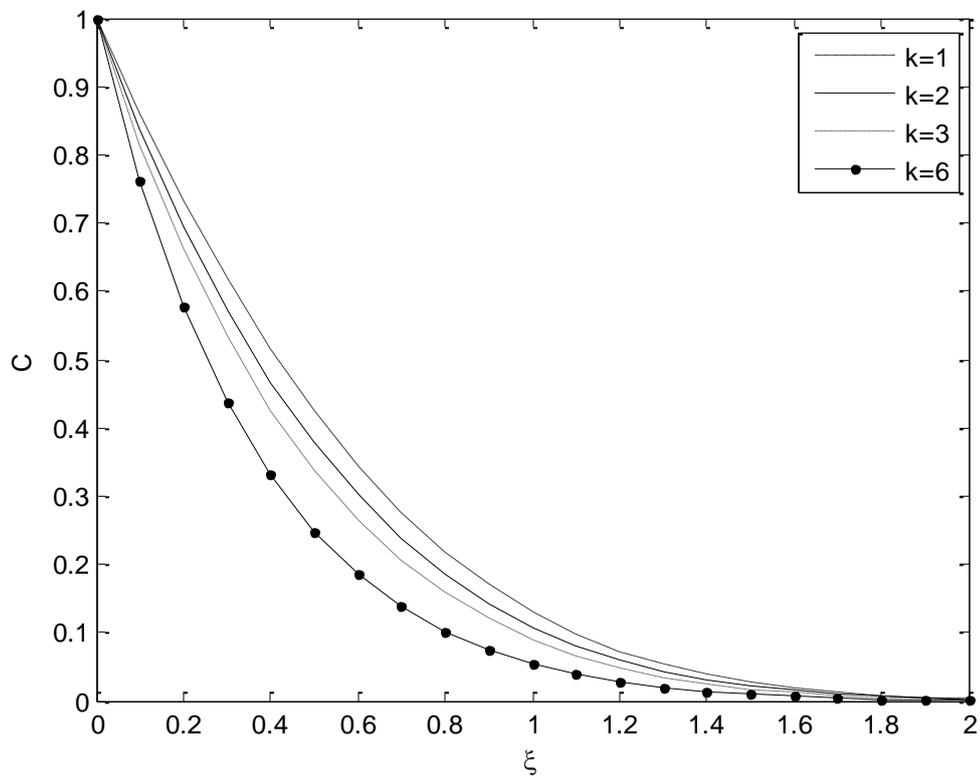


Fig 12. Variation of temperature for different values of Heat source parameter

When $Sc=0.4$, $k=5$, $M=1$, $t=0.5$, $Ko=0.1$.



**Fig 13. Variation of concentration for different values of
Chemical reaction parameter**

When $Sc=1, Q=2, M=1, t=0.3, Ko=0.1.$

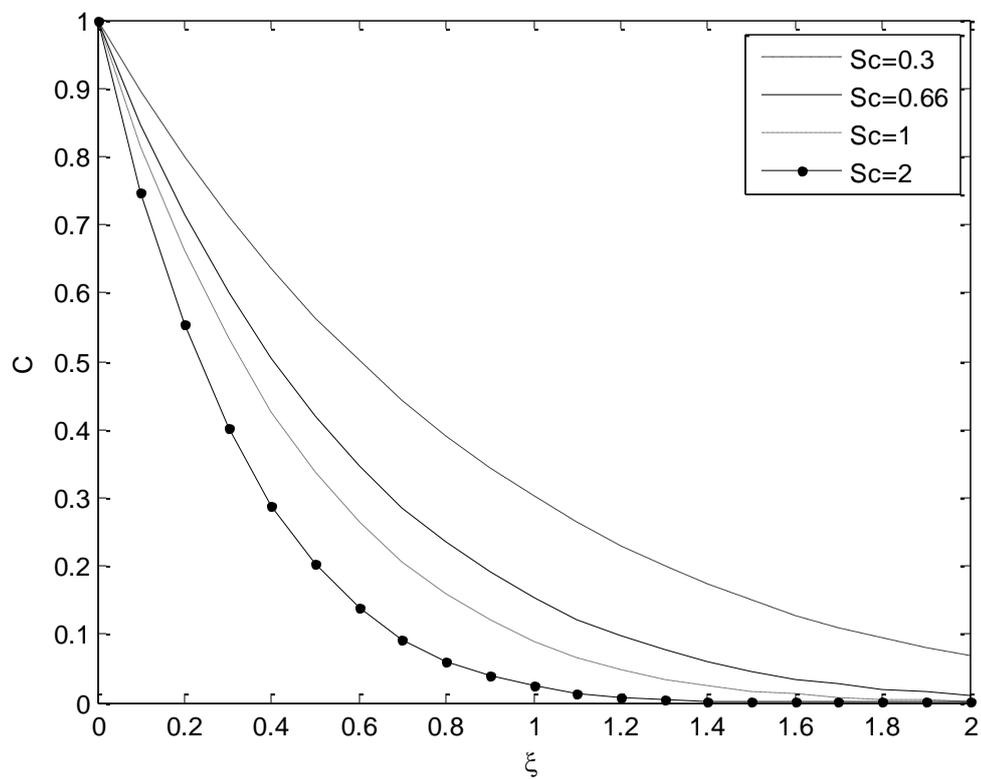


Fig 14. . Variation of concentration for different values of Schmidt number

When $Sc=1, K=3, Q=2, M=1, t=0.3, Ko=0.1$.

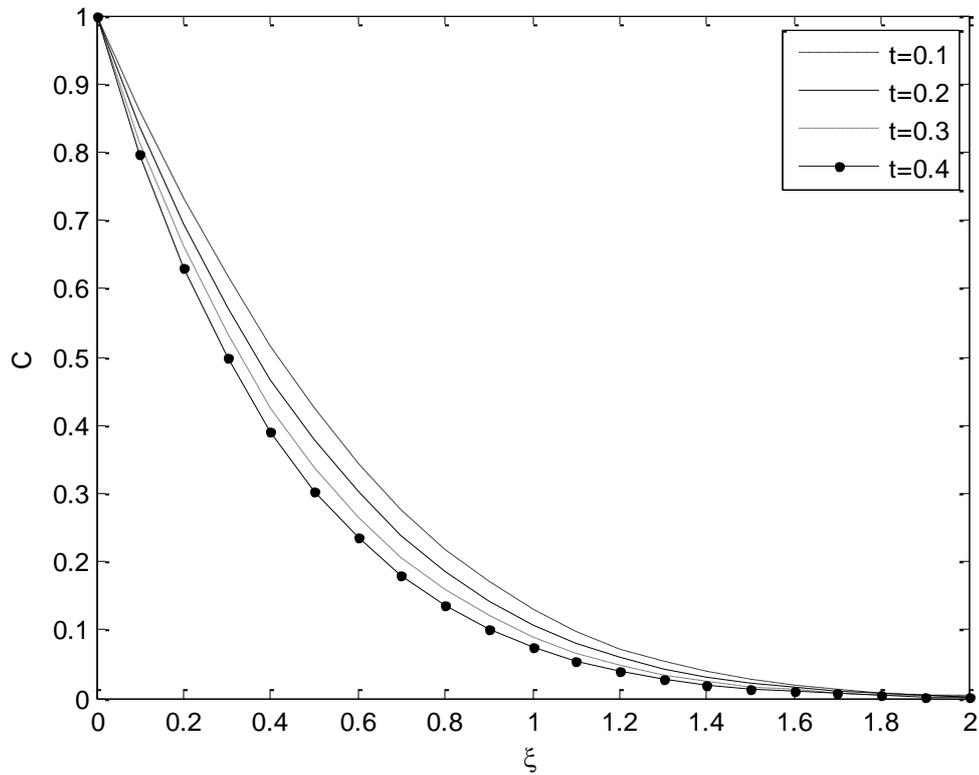


Fig 15. Variation of concentration for different values of time
When $Sc=1, K=3, Q=2, M=1, t=0.3, Ko=0.1.$

5 CONCLUSION

We can therefore conclude that:

1. For cooling of the plate by free convection current ($Gr > 0$)
 - (i). Increase in inclination angle α results in a decrease in the velocity
 - (ii). Increase in Q leads to increase in the velocity but in case of temperature it is

reversed.

(iii). Increase in the Chemical reaction results in a gradual decrease in the

Concentration profiles

2. For heating of the plate by free convection current ($Gr < 0$)

(i). the velocity profile shows flow reversal near the plate.

(ii). Increase in the magnetic field parameter M results in an increase in velocity.

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